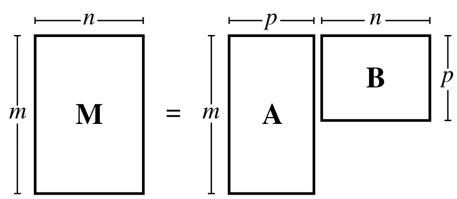
CS 322 Homework 1

out: Thursday 25 January 2007
due: Wednesday 31 January 2007

Consider the matrix–matrix product:

$$\mathbf{M} = \mathbf{A}\mathbf{B}$$

where $\mathbf{M}$ is $m \times n$, $\mathbf{A}$ is $m \times p$, and $\mathbf{B}$ is $p \times n$. Here is a picture:



One way of writing the definition of this is

$$m_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj} \quad \text{for } i = 1 \ldots m \text{ and } j = 1 \ldots n. \tag{1}$$

This corresponds to the MATLAB function:

```
M = multiply(A, B)

M = zeros(m,n);
for i = 1:m
   for j = 1:n
      for k = 1:p
         M(i,j) = M(i,j) + A(i,k) * B(k,j);
      end
   end
end
```

Instead of doing it this way, we could interpret the sum in (1) as a dot product between a row of A and a column of B. Let's introduce MATLAB-inspired names for the rows and columns:

$$\mathbf{A} = \begin{bmatrix} - \; \mathbf{a}_{1:} \; - \\ \vdots \\ - \; \mathbf{a}_{m:} \; - \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} | & & | \\ \mathbf{b}_{:1} & \cdots & \mathbf{b}_{:n} \\ | & & | \end{bmatrix}.$$

Now we can express elements of $\mathbf{M}$ more concisely:

$$m_{ij} = \mathbf{a}_{i:} \cdot \mathbf{b}_{:j} \quad \text{for } i = 1 \ldots m \text{ and } j = 1 \ldots n. \tag{2}$$

Or, in the same way we did in lecture, we can think of a column vector made up of dot products like this as a matrix product:

$$m_{:j} = \mathbf{A}\,\mathbf{b}_{:j} \quad \text{for } j = 1 \ldots n. \tag{3}$$

Problem 1: Find two more ways to express $\mathbf{M}$ using operations that operate on one vector at a time (dot products, vector addition, etc.) and write them down as equations similar to (2). Write up all three as MATLAB functions that each contain two nested loops.

Problem 2: Find two more ways to express $\mathbf{M}$ using operations that operate on a matrix at a time (Matrix–vector products, matrix addition, etc.) and write them down as equations similar to (3). Write up all three as MATLAB functions that each contain a single loop.

Print out your code and turn it in, with your equations, in lecture on the due date.