CS 322: Section Assignment 9

Week 9: March 27 - March 31

You may freely collaborate on section problems. They are not collected. Solutions are discussed in section and posted on the website. For maximum benefit, it is best to actually write MATLAB solutions and run them. Section problems and their variations can appear on course examinations. Problems are generally taken from Introduction to Scientific Computing-A Matrix-Vector Approach Using Matlab by C. Van Loan.

1. Suppose \( n \) is a given positive integer and that \( d \) is a given \( n \)-by-1 vector, \( B \) is a given \( n \)-by-\( n \) matrix, and \( A \) is a given nonsingular \( n \)-by-\( n \) matrix. Write an efficient MATLAB script that computes the scalar \( z \) defined by

\[
z = d^T B^T A^{-1} B d.
\]

Make effective use of the Matlab function \([L,U,P] = lu(A)\).

2. Gaussian elimination with pivoting is used to solve a 2-by-2 system \( Ax = b \) on a computer with machine precision \(10^{-16}\). It is known that \( \| A \| \| A^{-1} \| \approx 10^{10} \) and that the exact solution is given by

\[
x = \begin{bmatrix} 1.234567890123456 \\ 0.000123456789012 \end{bmatrix}.
\]

Underline the digits in \( x_1 \) and \( x_2 \) that probably agree with the corresponding digits in the computed solution. Explain the heuristic assumptions used to answer the question.

3. Assume that \( A \) is a given \( n \)-by-\( n \) nonsingular matrix and that \( b, c, \) and \( d \) are given column \( n \)-vectors. Write a MATLAB script that assigns to \( \alpha \) and \( \beta \) the scalars \( \alpha \) and \( \beta \) that minimize the 2-norm of the solution to the linear system \( Ax = b + \alpha c + \beta d \). Use \( \backslash \) for all linear system solving and all least squares minimization.

4. The Frobenius norm of a matrix \( C \in \mathbb{R}^{m \times n} \) is given by

\[
\| C \|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^2}
\]

Complete the following function so that it performs as specified

```matlab
function x = BestFitM(G,B)
% G is a k-by-1 cell array with each G{k} being an m-by-n matrix.
% B is an m-by-n matrix
% x is a k-by-1 vector with the property that the Frobenius norm of
% C = x(1)G{1} + \ldots + x(k)*G{k} - B is minimized.

% Hint. Look at the columns of C and turn the problem into a conventional min \| Ax - b \|_2 problem and solve via \backslash.
```

Complete the following function so that it performs as specified