CS 322: Section Assignment 8

Week 8: March 13 - 17

You may freely collaborate on section problems. They are not collected. Solutions are discussed in section and posted on the website. For maximum benefit, it is best to actually write MATLAB solutions and run them. Section problems and their variations can appear on course examinations. Problems are generally taken from Introduction to Scientific Computing-A Matrix-Vector Approach Using Matlab by C. Van Loan.

P1.

Assume that \( A \) and \( C \) are given \( n \)-by-\( n \) matrices and that \( A \) is nonsingular. Assume that \( g \) and \( h \) are given \( n \)-by-1 vectors. Write a MATLAB script that computes \( n \)-vectors \( y \) and \( z \) so that both of the following equations hold:

\[
Ay + Cz = g \\
Az = h.
\]

You may use the \( \backslash \) operator and \( \text{lu} \).

P2.

Assume that \( A \), \( B \), and \( C \) are nonsingular \( n \)-by-\( n \) matrices and that \( f \) is an \( n \)-by-1 vector. Write an efficient MATLAB fragment that computes a vector \( x \) so that \( ABCx = f \).

P3.

Assume that \( A \) is a given \( n \)-by-\( n \) nonsingular matrix and that \( b \) and \( c \) are given column \( n \)-vectors. Write a MATLAB script that computes a scalar \( \alpha \) (if possible) so that the solution to \( Ax = b + \alpha c \) satisfies \( x(1) = 0 \). Make effective use of \( \text{lu} \) and \( \backslash \). If it is not possible to chose \( \alpha \) so that \( x(1) = 0 \), then the script should print the message “impossible”.

P4.

Suppose \( T \in \mathbb{R}^{n \times n} \) and \( S \in \mathbb{R}^{m \times m} \) are given upper triangular matrices and that \( B \in \mathbb{R}^{m \times n} \). Write a MATLAB function \( X = \text{Sylvester}(S,T,B) \) that solves the matrix equation \( SX - XT = B \) for \( X \). Note that if we compare \( k \)th columns in this equation, we obtain

\[
SX(:,k) - \sum_{j=1}^{k} T(j,k)X(:,j) = B(:,k).
\]

That is,

\[
(S - T(k,k)I)X(:,k) = B(:,k) + \sum_{j=1}^{k-1} T(j,k)X(:,j).
\]

By using this equation for \( k = 1:n \), we can solve for \( X(:,1), \ldots, X(:,n) \) in turn. Moreover, the matrix \( S - T(k,k)I \) is upper triangular so that when \( \backslash \) is used to solve this system it requires \( m^2 \) flops. Assume that no diagonal entry of \( S \) is a diagonal entry of \( T \).