3. Since the \( n^{th} \) derivative of \( \cos(x) \) is either \( \sin(x) \), \( \cos(x) \), \( -\sin(x) \), or \( -\cos(x) \), we see that \( |f^{(n)}| \leq 1 \) for all \( x \in [-\pi, \pi] \). So

\[
|f(x) - p_{n-1}(x)| \leq \frac{1}{4n} \left( \frac{\pi - (-\pi)}{n - 1} \right)^n = \frac{1}{4n} \left( \frac{2\pi}{n - 1} \right)^n
\]

From above, we see that as \( n \to \infty \), \( |f(x) - p_{n-1}(x)| \to 0 \).

To choose \( n \) in order to have error less than \( 10^{-5} \), observe that \( \frac{1}{4(12)} \left( \frac{2\pi}{12-1} \right)^{12} = 2.5131e-5 \), and \( \frac{1}{4(13)} \left( \frac{2\pi}{13-1} \right)^{13} = 4.2754e-6 \). Hence, you need \( n \) to be at least 13. Below is the code used to plot

\[
\%
\%
\text{Plot } f(x) - p_{n-1}(x) \text{ in the interval } [-\pi, \pi],
\%
\text{where } f(x) = \cos(x) \text{ and } p_{n-1}(x) \text{ is the } n \text{ points}
\%
\text{interpolant of } f(x). \ n \text{ is set to be 13, as found}
\%
\text{previously.}
\%
\%
\text{n=13;}
\text{u=0; v=1;}
\text{x=linspace(-pi,pi,n)'};
\text{a=InterpV(x,cos(x),u,v)};
\text{z=linspace(-pi,pi)'};
\text{ytrue=cos(z)};
\text{y=HornerV(a,z,u,v)};
\text{plot(z,abs(ytrue-y))};
\text{title('Error of interpolant using n=13')};
\%
\% Plot f(x) - p_{n-1}(x) in the interval [-pi,pi],
\% where f(x) = \cos(x) and p_{n-1}(x) is the n points
\% interpolant of f(x). n is set to be 13, as found
\% previously.
\%
\%
\text{n=13;}
\text{u=0; v=1;}
\text{x=linspace(-pi,pi,n)'};
\text{a=InterpV(x,cos(x),u,v)};
\text{z=linspace(-pi,pi)'};
\text{ytrue=cos(z)};
\text{y=HornerV(a,z,u,v)};
\text{plot(z,abs(ytrue-y))};
\text{title('Error of interpolant using n=13')};
\%
\% Plot f(x) = 1/(1+25*x^2) and its interpolant using
\% 5, 11, 15, and 21 equally-spaced points.
\%
\%
\text{n=13;}
\text{u=0; v=1;}
\text{x=linspace(-pi,pi,n)'};
\text{a=InterpV(x,1/(1+25*x^2),u,v)};
\text{z=linspace(-pi,pi)'};
\text{ytrue=1/(1+25*z^2)};
\text{y=HornerV(a,z,u,v)};
\text{plot(z,abs(ytrue-y))};
\text{title('Error of interpolant using n=13')};
Opposite to what one expected from question 3, increasing number of interpolation points increases the error near the left and right endpoints of the interval. Let’s try using Chebyshev points

```matlab
% close all
z=linspace(-1,1)';
ytrue=1./(1+25.*z.^2);
for n=[5 11 15 21]
    x=linspace(-1,1,n)';
    a=InterpV(x,1./(1+25*x.^2));
    y=HornerV(a,z);
    figure
    plot(z,ytrue,':',z,y,'-');
    legend('function','interpolant');
    title(['n=',num2str(n)]);
end
```

Opposite to what one expected from question 3, increasing number of interpolation points increases the error near the left and right endpoints of the interval. Let’s try using Chebyshev points.
Figure 2: Question 4: evenly spaced points, n=5

Figure 3: Question 4: evenly spaced points, n=11
Figure 4: Question 4: evenly spaced points, n=15

Figure 5: Question 4: evenly spaced points, n=21
When Chebyshev points are used, increasing the number of interpolation points does decrease the error.
Figure 7: Question 4: Chebyshev points, n=11

Figure 8: Question 4: Chebyshev points, n=15
Figure 9: Question 4: Chebyshev points, n=21