3. Since the $n^{th}$ derivative of $cos(x)$ is either $sin(x)$, $cos(x)$, $-sin(x)$, or $-cos(x)$, we see that $|f^{(n)}| \leq 1$ for all $x \in [-\pi, \pi]$. So

$$\begin{aligned} |f(x) - p_{n-1}(x)| &\leq \frac{1}{4n}(\frac{\pi - (-\pi))}{n-1})^n \\ &= \frac{1}{4n}(\frac{2\pi}{n-1})^n \end{aligned}$$

From above, we see that as $n \to \infty$, $|f(x) - p_{n-1}(x)| \to 0$.

To choose $n$ in order to have error less than $10^{-5}$, observe that $\frac{1}{4(12)}(\frac{2\pi}{12-1})^{12} = 2.5131e - 5$, and $\frac{1}{4(13)}(\frac{2\pi}{13-1})^{13} = 4.2754e - 6$. Hence, you need $n$ to be at least 13. Below is the code used to plot

```
%
% Plot f(x) - p_{n-1}(x) in the interval [-pi,pi],
% where f(x) = cos(x) and p_{n-1}(x) is the n points
% interpolant of f(x). n is set to be 13, as found
% previously.
%
n=13;
u=0; v=1;
x=linspace(-pi,pi,n)';
a=InterpV(x,cos(x),u,v);
z=linspace(-pi,pi)';
ytrue=cos(z);
y=HornerV(a,z,u,v);
plot(z,abs(ytrue-y));
title('Error of interpolant using n=13');
```

As seen from figure 1, the actual error is less than $10^{-5}$ as expected.

4. Below is the script for evenly spaced points case

```
%
% Plot f(x) = 1/(1+25*x^2) and its interpolant using
% 5,11,15, and 21 equally-spaced points.
```

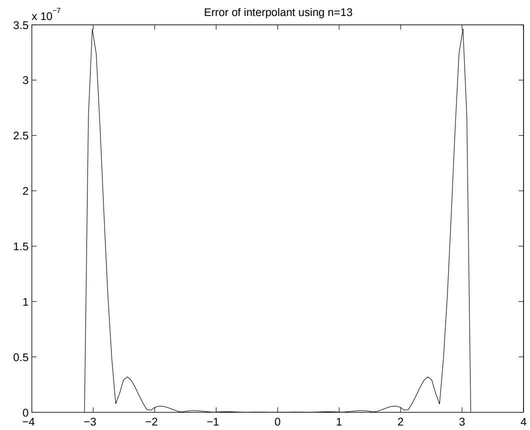


Figure 1: Question 3: Error of interpolant using n=13

```
%
close all
z=linspace(-1,1)';
ytrue=1./(1+25.*z.^2);
for n=[5 11 15 21]
    x=linspace(-1,1,n)';
    a=InterpV(x,1./(1+25*x.^2));
    y=HornerV(a,z);
    figure
    plot(z,ytrue,':',z,y,'-');
    legend('function','interpolant');
    title(['n=',num2str(n)]);
end
```

Opposite to what one expected from question 3, increasing number of interpolation points increases the error near the left and right endpoints of the interval. Let's try using Chebyshev points

```
%
% Plot f(x) = 1/(1+25*x^2) and its interpolant using
% 5,11,15, and 21 Chebyshev points.
%
```

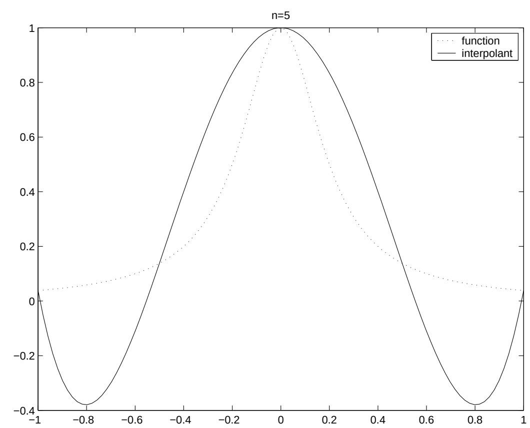


Figure 2: Question 4: evenly spaced points, n=5

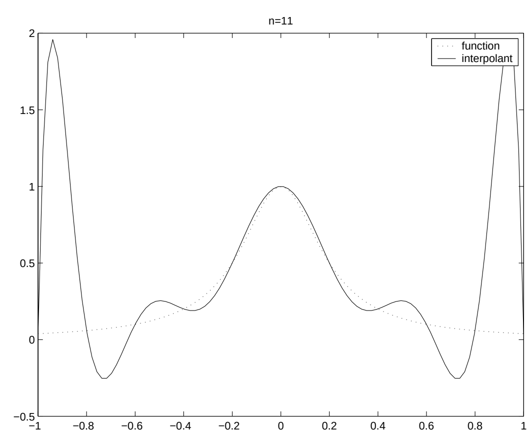


Figure 3: Question 4: evenly spaced points, n=11

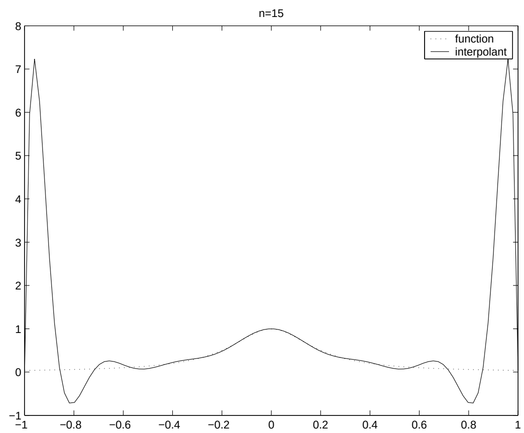


Figure 4: Question 4: evenly spaced points, n=15

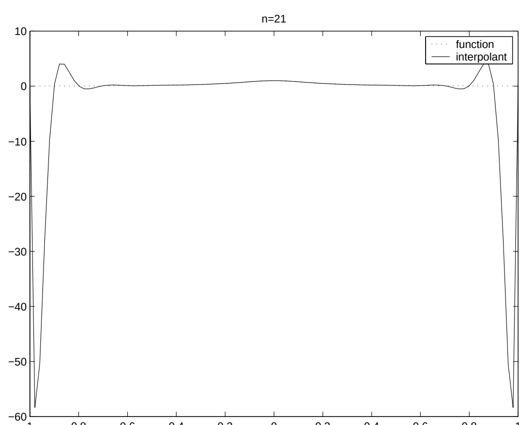


Figure 5: Question 4: evenly spaced points, n=21

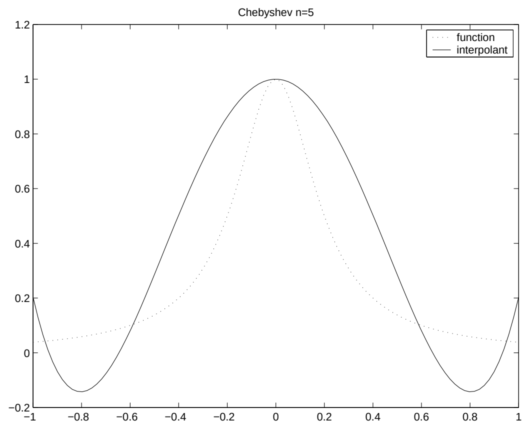


Figure 6: Question 4: Chebyshev points, n=5

```
close all
z=linspace(-1,1)';
ytrue=1./(1+25.*z.^2);
for n=[5 11 15 21]
    x = -cos((2*(1:n)-1)*pi/2/n)';
    a=InterpV(x,1./(1+25*x.^2));
    y=HornerV(a,z);
    figure
    plot(z,ytrue,':',z,y,'-');
    legend('function','interpolant');
    title(['Chebyshev n=',num2str(n)]);
end
```

When Chebyshev points are used, increasing the number of interpolation points does decrease the error.

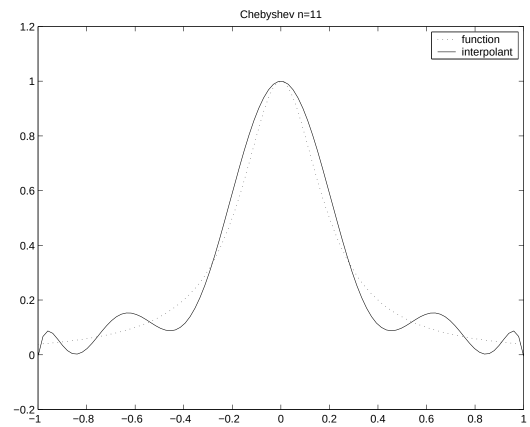


Figure 7: Question 4: Chebyshev points, n=11

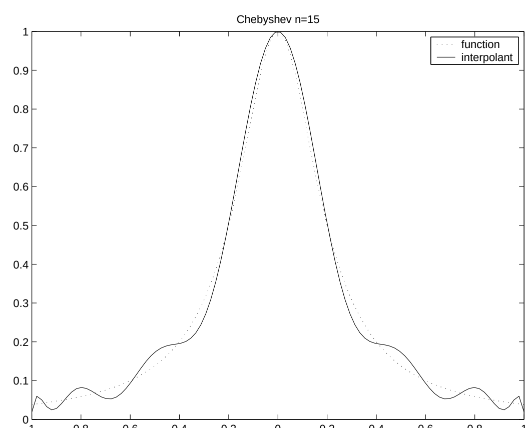


Figure 8: Question 4: Chebyshev points, n=15

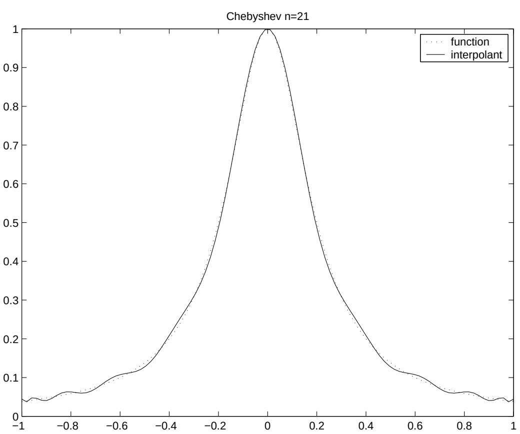


Figure 9: Question 4: Chebyshev points, n=21