**Problem set 1 question 1** (25 points)

To solve for the coefficients in the Newton representation using InterpN.m, we see that during pass $k$ through the loop $2(n - k)$ subtractions and $(n - k)$ divisions are performed. So the total number of flops for this algorithm is

$$
\sum_{k=1}^{n-1} (2(n - k) + (n - k)) = \sum_{k=1}^{n-1} 3n - 3k = 3n(n - 1) - 3 \sum_{k=1}^{n-1} k
$$

$$
= 3n^2 - 3n - 3(n-1)(n)/2 = \frac{3}{2}n^2 - \frac{3}{2}n \approx \frac{3}{2}n^2
$$

HornerN evaluates the Newton representation polynomial with one subtraction, one multiplication, and one addition per degree of the polynomial. Hence the cost is $3(n - 1)$ flops to evaluate the polynomial interpolant of $n$ points. So the Newton method costs $\frac{3}{2}n^2$ flops (to leading term) to compute the coefficients and evaluate the polynomial once.

To evaluate the interpolant in the Lagrange representation, we see that for each of the $n$ terms we have $2(n - 1)$ subtractions (half in the numerator and half in the denominator), $2(n - 2) + 1$ multiplications ($(n - 2)$ in the numerator, $(n - 2)$ in the denominator and 1 to multiply by $y_i$), and one division. So the total number of flops is

$$
n(2(n-1) + 2(n-2) + 1 + 1) = n(4n - 4) = 4n^2 - 4n.
$$

So solving for and evaluating the polynomial is less expensive in the Newton representation since Newton costs about $\frac{3}{2}n^2$ and Lagrange costs about $4n^2$. You could certainly save some flops in the Lagrange method, but it would still be an $O(n^2)$ calculation. To calculate the coefficients and evaluate the polynomial at $n$ points would cost $\frac{3}{2}n^2 + n \times (3n - 3) \approx \frac{9}{2}n^2$ flops using the Newton representation, and $n \times (4n^2 - 4n) \approx 4n^3$ flops in the Lagrange representation. To evaluate at $n^2$ points would cost approximately $\frac{3}{2}n^2 + n^2 \times (3n - 3) \approx 3n^3$ flops using the Newton representation, and $n^2 \times (4n^2 - 4n) \approx 4n^4$ flops in the Lagrange representation. So although the Lagrange representation is elegant, it is less attractive (computationally) than the Newton representation.