Quiz #2
Due in class, Friday, July 18, 2003

Have We Met Before?

Answer all below questions. You may consult only the TA, but he has been struck with a case of laryngitis and should not use his voice. Submit all scratch paper and full, clear reasoning; this will help us give you partial credit. Note that by flops we mean only additions, subtractions, multiplications and divisions. Assume that both A and B, given below, are nonsingular. Use only higher order terms for running times of LU factoring, forward and backward substitution. The bonus is worth little. It is mostly to give you food for thought and provide more chances to demonstrate mastery of the subject.

1) Suppose we wish to find x such that $A^m\underline{x} = \underline{b}$, $(m \geq 2)$, where A^m is the matrix product of A with itself m times.

a) Show how forming $B = A^m$ and then solving $B\underline{x} = \underline{b}$, results in

$$(2m - \frac{4}{3})n^3 - (m-3)n^2$$

flops. Note: form B in the straightforward, brute-force manner, requiring $m-1$ matrix multiplication, i.e. by calculating A^2, then $A^3 = A^2A$, and so on. While it is true that one could form it in an alternative way that would require fewer flops, this optimization is not of our interest in this quiz.

b) Following the "LU Mentality", show how to solve the system without forming B and that this neccessitates only $\frac{2}{3}n^3 + 2mn^2$ flops.

Bonus: Under what conditions on m will it be more efficient to form B and solve $B\underline{x} = \underline{b}$ than to carry out the method in (b)?