Midterm

Friday, July 25, 2003

Answer all below questions. You may consult only the proctor. Submit all scratch paper and full, clear reasoning; this will help us give you partial credit.

1) (34 pts) You have the three data points

$$(x_1,\, f(x_1)) = (0,\, 0),$$

$$(x_2,\, f(x_2)) = (\frac{\pi}{2},\, 1)$$

and

$$(x_3,\, f(x_3)) = (\pi,\, 0),$$

where f is the alledged underlying unknown function. Using the representation

$$p_{n-1}(x) = a_1 + a_2 x + \cdots + a_n x^{n-1},$$

set up, but do not solve, the equations that give the a_i's so that p_{n-1} interpolates f at the three data points. Design a 3-point interpolation function for this data based on the representation

$$p_{n-1}(x) = a_1 + a_2\left(\frac{x}{\pi}\right) + a_3\left(\frac{x}{\pi}\right)^2 + \cdots + a_n\left(\frac{x}{\pi}\right)^{n-1}.$$

Be sure to explicitly write down your function. Explain your method's virtues, drawbacks.

Bonus: If possible, give interpretations of the unknowns in your method.

2) (33 pts) Clearly delineate how to find x given that

$$x = b^T C^{-1} b,$$

where $C = AA^T$, matrices A and C are nonsingular, as efficiently as possible. Assume that pivoting is not necessary. What does the cost of your method boil down to? Suppose we formed C, formed C^{-1}, then x, and we called this method 2. What would be the savings from your method, given above, over method 2 (Assume that forming C^{-1} from C requires $\frac{8}{3}n^3$)? Note that forming C^{-1} from C is usually unstable and therefore a bad idea, but we consider it for sake of comparison.

3a) (23 pts) Find w_i, x_i, so that the integration formula

$$\int_{-1}^{1} x^2 f(x)dx = w_1 f(x_1) + w_2 f(x_2)$$

is exact for all $f \in \Pi_3$ (Hint: following the methodology in the text is one way to do it.).

3b) (10 pts) Let

$$I_1 = \frac{1}{3} \int_a^b (f - g - h)dx$$

and

$$I_2 = \frac{1}{3} \int_a^b (f + g + h)dx.$$

Clearly show how to estimate I_i ($i = 1, 2$) to within a tolerance of tol using Matlab's `quad` function. Show why your approximations serve their purpose, clearly stating any assumptions you are using.

Bonus. Discuss two more reasons your method wins over a brute-force method.