Homework #3
Due Thursday, July 24, 2003

What's In A Name?

Answer all questions. Submit any pertinent hand calculations and/or Matlab code in a neat manner in order to help us give you partial credit. Readings: Van Loan 4.0-4.4. Code from the text is available at www.cs.cornell.edu/cv, click on "M-files".

1) Do problem P4.1.4 in the text. What is the moral of the story?

2) a) Show that

$$I = \int_0^1 \frac{1}{\sqrt{x}} dx < \infty$$

analytically, by finding $I$.

b) Approximate $I$ using the closed Newton-Cotes rule for $m = 3$. What goes wrong? How can you get around this?

c) All error bounds for Newton-Cotes rules are of the form

$$\left| Q_{NC(m)} - \int_a^b f(x)dx \right| \leq kM(\frac{b-a}{m \pm \alpha})^{m \pm \beta}, \tag{1}$$

where $k$ is some positive constant,$\alpha \geq 1$ and $\beta \geq 1$ are some integers and $M$ is such that

$$\left| f^{(n)}(x) \right| \leq M$$

for any $x \in [a, b]$ and $n$ is some integer. Tell why in this case (1) is useless. Explicit calculations are needed here.

d) Augment the text code (not actually in the text but on its website) *ShowNCOpenError* so that it computes $I$ and its error. You need not worry about the error bound.

3) You are the engineer in charge of computing the integrals,

$$I_i = \int_a^b (\alpha_i g(x) + \beta_i h(x))dx,$$

for $i = 1, 2, \ldots, m$, to within the specified tolerance of $tol$. Suppose that computing functional values of $g$ is very expensive and similarly for $h$.

a) Suppose the expense of computing $g$ and/or $h$ limits you so that you could only compute one approximation to $g_I = \int_a^b g(x)dx$ and one approximation to

$h_I = \int_a^b h(x)dx$. Why is this reasonable? Show why it is possible to approximate all $I_i$ to within the specified tolerance, *tol*, since, for example with the Newton-Cotes rules, we can approximate $g_I$ to within a tolerance of $\frac{tol}{2M_\alpha}$, where $M_\alpha = \max\{|\alpha_1|, \ldots, |\alpha_m|\}$.

b) You could use Newton-Cotes formulas to compute $\int_a^b f_i(x)dx$ to the specified tolerance of *tol*, where $f_i = \alpha_i g + \beta_i h$, for $i = 1, 2, \ldots, m$. Considering part (a), what are disadvantages of doing this? Give quantitative evidence to support your claims.

4) TBA