CS322
Homework #1
Due Friday, July 11, 2003

In the Beginning...

Answer all questions. Submit any pertinent hand calculations and/or Matlab code in a neat manner in order to help us give you partial credit. Readings: Van Loan 2.0-2.3, 3.0-3.3.

1) Describe briefly an example of each of the following:
a) A numerical method that is inefficient.
b) A numerical method that is inaccurate.

2) Describe briefly (a sentence or two for each) three topics that will be covered in this class. The text will be helpful here.

3) *A numerical analyst's answer to "hello world".* Let $f(x) = \pi \sin 3x + e^{-3} \cos 2x$ on the interval $[0,\, 3\pi]$ and $n = 3$. Let V_n and N_n denote the Vandermonde and Newton representations of polynomial interpolants of f, respectively. (a) Should V_n be equal to N_n for all $x \in [0,\, 3\pi]$? (b) Why or why not?

Let g be some function defined on the interval I and let p be some interpolant of g. Denote by ae the *absolute error* and define ae by

$$ae(x) = |g(x) - p(x)|\,,$$

where x is any point in I.

(c) Let $\underline{x} = (x_1,\, x_2,\, x_3) = (0,\, \frac{3\pi}{2},\, 3\pi)$ and $y_i = f(x_i)$, for $i = 1,\, 2,\, 3$. Find $ae(2\pi)$ for our f using both V_n and N_n. I worked out the following crude upper bound for f's third derivative,

$$\left| f^{(3)}(\eta) \right| \leq 85.168,$$

for some $\eta \in [0,\, 3\pi]$, (d) are your observed ae's smaller than our theoretical bound?

4) *It's All About Who Represents You.* Recall the cubic representation given in the text:

if

$$q(z) = a + b(z - x_L) + c(z - x_L)^2 + d(z - x_L)^2(z - x_R),$$

then

$$\begin{aligned} q(x_L) &= y_L, \quad & q(x_R) &= y_R, \\ q'(x_L) &= s_L, \quad & q'(x_R) &= s_R. \end{aligned}$$

Consider the following, alternative representation, that has the same conclusion:

if

$$q_c(z) = A + Bz + Cz^2 + Dz^3,$$

then

$$\begin{aligned} q_c(x_L) &= y_L, \quad q_c(x_R) = y_R, \\ q_c'(x_L) &= s_L, \quad q_c'(x_R) = s_R. \end{aligned}$$

(a) Write down the matrix equations that result from this alternative. (b) Find D, thereby showing that

$$D = \frac{(x_L - x_R)(s_L + s_R) - 2(y_L - y_R)}{(x_L - x_R)^3}.$$

Consider how much work it took you to find D, and in at most two sentences (c) comment on the virtues of the original cubic representation. (d) Where else have we seen this virtue (in the readings and/or lecture)?