CS 322: Prelim 1 Review Questions

Solutions posted on the course website at noon, Monday, March 11. These problems are not necessarily typical exam problems. The exam syllabus is defined by S1-S7, P1-P3, and all readings through §6.2.

1. Assume that $M$ is a positive integer and that $M$ and $M + 2$ are floating point numbers in a base-2 floating point number system with $t$-bit mantissas. If $M + 1$ is not a floating point number then what can you say about $t$?

2. Complete the following function so that it performs as advertised. Be sure to vectorize.

   ```matlab
   function t = Prod(x)
   % x is a column n-vector and n > 3.
   % y = A*x where A is the n-by-n matrix defined by
   %
   % 0       if |i-j|>2
   % A(i,j) =
   % cos(|i-j|)*pi/n  if |i-j|<=2
   ```

3. Complete the following function so that it performs as advertised. Be sure to vectorize.

   ```matlab
   function m = MaxJump(x,y)
   % x and y are column n-vectors and x(1) < x(2) < ... < x(n).
   % Let S be the cubic spline produced by spline(x,y).
   % m is the maximum value of f(x) on the interval [x(2), x(n-1)] where
   % f(x) is the limit of |S'''(x+delta) - S'''(x-delta)| as delta goes to zero.
   ```

4. Complete the following function so that it performs as advertised.

   ```matlab
   function c = Convert(a,alfa,beta)
   % a is a column 4-vector and alfa and beta are scalars.
   % c is a column 4-vector so that the cubic polynomials
   %
   % p(x) = a(1) + a(2)(x-alfa) + a(3)(x-alfa)^2 + a(4)(x-alfa)^3
   % q(x) = c(1) + c(2)(x-beta) + c(3)(x-beta)^2 + c(4)(x-beta)^3
   % are identical.
   ```

5. Modify the following function so that it takes full advantage of $A$’s banded structure.

   ```matlab
   function [L,U] = SpecLU(A,p)
   % A is an n-by-n matrix with lower bandwidth 1 and upper bandwidth p.
   % Assume that A has an LU factorization.
   % Computes the factorization A = LU where L is an n-by-n unit lower unit
   % triangular matrix and U is an n-by-n upper triangular.
   
   [n,n] = size(A); v = zeros(n,1);
   for k=1:n-1
   v(k+1) = A(k+1,k)/A(k,k);
   A(k+1,k:n) = A(k+1,k:n) - v(k+1)*A(k,k:n);
   end
   L = diag(v(2:n),-1); U = triu(A);
   ```

6. Assume that the function $y = f(x)$ is available and that $f$ has period $\pi$. How would you use QUAD to compute

   \[ I = \int_a^b f(x)dx \]

   with absolute error $\leq 10^{-4}$?