Problem 1
Assume you are sampling values from a some unknown distribution with finite \( \mu \) and \( \sigma \) (say a survey asking students to grade their TA on a 0-100 scale).
After getting the first 10 responses (10 samples) you got a mean of \( \bar{X}_{10} = 61 \).
Give a 95% confidence interval for \( \mu \).
After receiving all 50 responses from the students you got a mean of \( \bar{X}_{50} = 53 \).
Give a 95% confidence interval for \( \mu \).
Why would you use 2 different distributions for the two problems?

Problem 2
You have seen in class the maximum likelihood estimator \( \hat{A} = \max(x_i) \) for the parameter \( A \) of a Uniform distribution on the interval \((0, A)\).
Find a maximum likelihood estimator for the parameters \( A \) and \( B \) of a Uniform distribution on the interval \((A, B)\)?
Find a maximum likelihood estimator for the parameter \( A \) of a Uniform distribution on the interval \((-A, A)\).
Find a maximum likelihood estimator for the parameter \( A \) of a Uniform distribution on the interval \((A, 2A)\).

Problem 3
Estimator \( \hat{\Theta} \) is called an unbiased estimator for \( \Theta \) if \( E(\hat{\Theta}) = \Theta \) (notice that \( \hat{\Theta} \) is indeed a random variable!).
Consider a Uniform distribution on the interval \((0, A)\).
Is the maximum likelihood estimator for \( A \) unbiased?
Is \( \hat{A}_1 = 2\bar{X}_n \) an estimator for \( A \)? Is it a reasonable estimator for \( A \)?
Is the above defined \( \hat{A}_1 \) an unbiased estimator for \( A \)?
Is \( \hat{A}_2 = 2 \) an estimator for \( A \)? Is it a reasonable estimator for \( A \)?
Is the above defined \( \hat{A}_2 \) an unbiased estimator for \( A \)?

Problem 4
Estimator \( \hat{\Theta}_1 \) is considered a tighter estimator for \( \Theta \) than the estimator \( \hat{\Theta}_2 \) if for any value of \( \Theta \) and any sequence of samples \( x_1, x_2, \ldots, x_n \) \( |\hat{\Theta}_1 - \Theta| < |\hat{\Theta}_2 - \Theta| \).
Once again consider a Uniform distribution on the interval \((0, A)\).
Is the maximum likelihood estimator for \( A \) tighter than the above defined \( \hat{A}_1 \)?
Is the maximum likelihood estimator for \( A \) tighter than the above defined \( \hat{A}_2 \)?
Can you find an estimator for \( A \) that is tighter than \( \hat{A}_1 \)?