Problem 1
A radioactive substance $X$ decays continuously at a constant rate of $3 \times 10^{-3}\%$ per year. What is the probability for a single substance $X$ atom to decay in the next 100 years?

Solution 1
With a constant rate of decay we can express the change in the number of atoms of substance $X$ in a sample of starting size $n_0$ as $-dN = N \cdot \lambda \cdot dt$, where $\lambda = 3 \times 10^{-3}\%$ per year (or just $\lambda = 3 \times 10^{-5}$ year$^{-1}$).
Like in class, here $N$ is the number of atoms (that didn’t decay yet) and $dN$ is the loss of substance $X$ atoms.
From the above equation we can get that $-\frac{dN}{N} = \lambda dt$.
We will integrate both sides. Notice that the left side our bounderies are $n_0$ and $n_T$ (initial sample size and sample size at time $T$) while on the right side our bounderies are 0 and $T$.

$$\int_{n_0}^{n_T} \frac{-dN}{N} = \int_{0}^{T} \lambda dt \Rightarrow \int_{n_0}^{n_T} \frac{1}{N} dN = \int_{0}^{T} -\lambda dt \Rightarrow \log(N)_{n_0}^{n_T} = -\lambda T \Rightarrow$$

$\log(n_T) - \log(n_0) = -\lambda(T-0) \Rightarrow \log\left(\frac{n_T}{n_0}\right) = -\lambda T \Rightarrow \frac{n_T}{n_0} = e^{-\lambda T} \Rightarrow n_T = n_0 e^{-\lambda T}$

We are looking for the amount of atoms that decayed which is

$$n_0 - n_T = n_0 - n_0 e^{-\lambda T} = n_0 (1 - e^{-\lambda T})$$

The fraction of the sample decayed in time $T$ can be expressed as

$$\frac{n_0 - n_T}{n_0} = \frac{n_0 (1 - e^{-\lambda T})}{n_0} = (1 - e^{-\lambda T})$$

Using the values from the question we get that in $T = 100$ years and a constant decay rate of $\lambda = 3 \times 10^{-5}$ year$^{-1}$ the fraction that decays is $(1 - e^{-3 \times 10^{-5} \times 100})$. So the probability of a single atom from the sample to decay in this time is $(1 - e^{-3 \times 10^{-3}}) \approx -3 \times 10^{-3}$. 
Problem 2
Using a Uniform random variable $U$ on the interval $(0, 10)$, how would you express
The Uniform random variable $Y$ on the interval $(-1, 1)$?
The Exponential random variable $W$ with parameter $\lambda = 2$?

Solution 2
If $U$ is Uniform on $(0, 10)$ then $\frac{U}{10}$ is Uniform on $(0, 2)$ and $Y = \frac{U}{5} - 1$ is Uniform on $(-1, 1)$.
We saw in section that if $F$ is a c.d.f and there exists an inverse function $F^{-1}$
then for a Uniform $(0, 1)$ random variable $V$, the new formed random variable
$X = F^{-1}(V)$ has the p.d.f. $F$.
In this problem we are interested in a r.v. $W$ with the c.d.f. $F_W(w) = 1 - e^{\lambda w} = 1 - e^{2w}$.
The function $F^{-1}$ would then be $\frac{-\log(1 - V)}{\lambda} = \frac{-\log(1 - V)}{2}$.
But $U$ is Uniform $(0, 10)$, so $V = \frac{U}{10}$ should be Uniform $(0, 1)$ and our desired random variable will be
$$W = \frac{-\log(1 - \frac{U}{10})}{2}$$