Homework 3

It is possible to use a minimization algorithm to compute the highest eigenvalue (and its corresponding eigenvector) of a matrix $A$.

Consider the target function $T = x'Ax$ for optimization where $x$ is the vector that we wish to determine. Design a conjugate gradient algorithm that determines the eigenvector with the highest eigenvalue using $T$ and the constraint that $x'x = 1$.

Write a Matlab code to perform the calculations. Do not use the conjugate gradient facility of Matlab.

What are the conditions on the matrix $A$?

Consider the matrices below. Only one of the matrices satisfies the conditions we must set in order to use $T = x'Ax$ optimization to determine the highest eigenvalue. Determine the matrix and explain your choice. Compute the highest eigenvalue using your code. In your report include the history of convergence (i.e. plot the value of the eigenvalue as a function of the optimization step).

$$
A_1 = \begin{pmatrix}
1 & 0.1 & 0 \\
0.1 & 1 & 0.2 \\
0 & 0.2 & -1
\end{pmatrix} \\
A_2 = \begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{pmatrix} \\
A_2 = \begin{pmatrix}
-3 & 1 & 0 \\
1 & -3 & 1 \\
0 & 1 & -4
\end{pmatrix}
$$