

## Lecture 17

# Physics in Games

# Warm-Up Activity

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- Think of a simple *physics-based mechanic*
  - Does not have to be novel
  - But should involve your character/avatar
- What *information* do you need to support it?
  - **Examples:** Mass, friction, volume
- What support do you need from the *designer*?
  - How do you “annotate” the art assets?
  - How does this affect the level editor?

# The Pedagogical Problem

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- Physics simulation is a **very** complex topic
  - No way I can address this in a few lectures
  - Could spend an entire course talking about it
  - **CS 5643**: Physically Based Animation
- This is why we have **physics engines**
  - Libraries that handle most of the dirty work
  - But you have to understand how they work
  - **Examples**: Box2D, Bullet, PhysX

# Approaching the Problem

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- Want to start with the **problem description**
  - Squirrel Eiserloh's *Problem Overview* slides
  - <http://www.essentialmath.com/tutorial.htm>
- Will help you understand the Engine APIs
  - Understand the limitations of physics engines
  - Learn where to go for other solutions
- Will cover Box2D API next time in depth

# Physics in Games

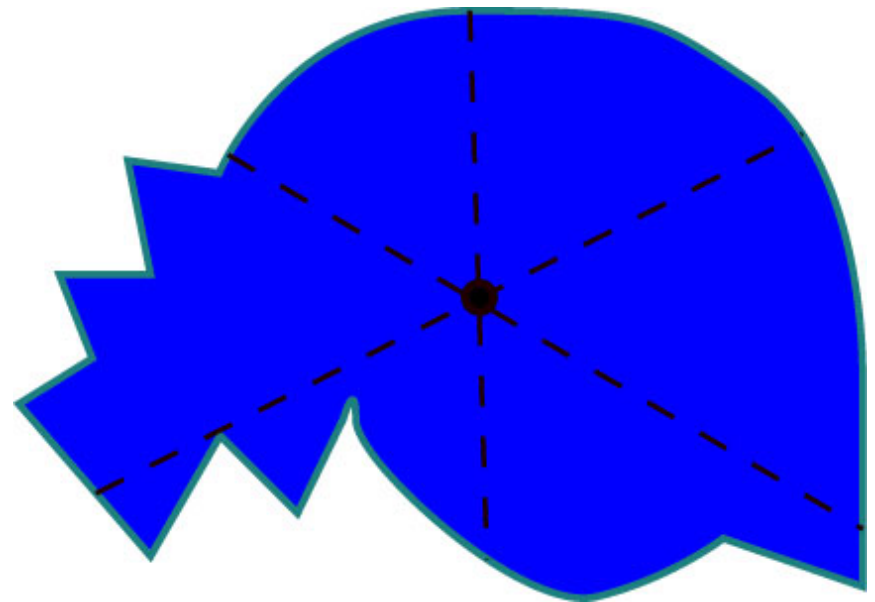
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- **Moving** objects about the screen
  - **Kinematics**: Motion ignoring external forces  
(Only consider position, velocity, acceleration)
  - **Dynamics**: The effect of forces on the screen
- **Collisions** between objects
  - **Collision Detection**: Did a collision occur?
  - **Collision Resolution**: What do we do?

# Motion: Modeling Objects

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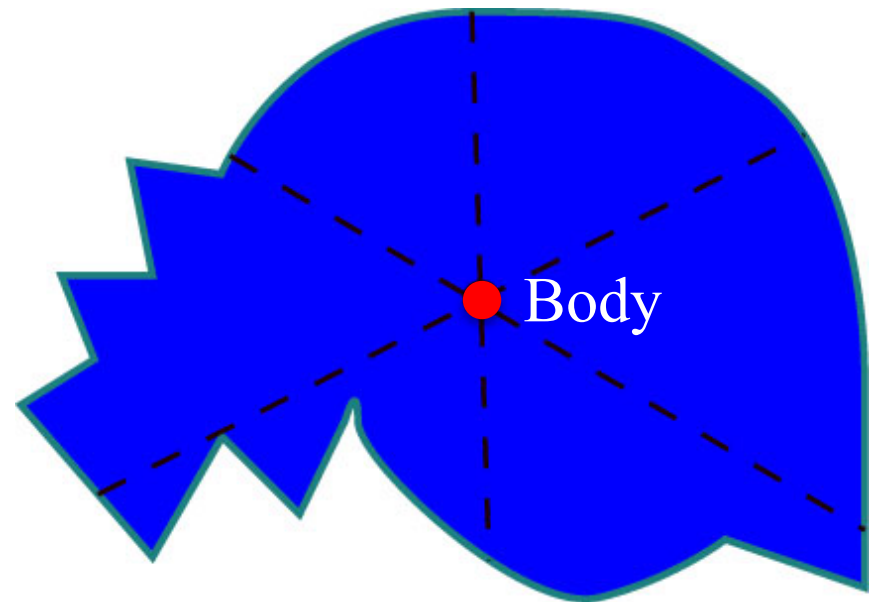
- Typically ignore **geometry**
  - Don't worry about shape
  - Only needed for *collisions*
- Every object is a **point**
  - *Centroid*: average of points
  - Also called: *center of mass*
  - Same if density uniform
- Use **rigid body** if needed
  - Multiple points together
  - Moving one moves them all



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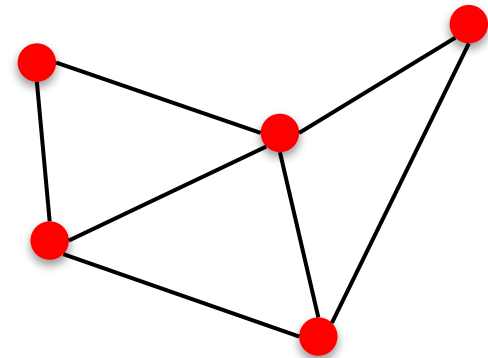


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Rigid Body





# Time-Stepped Simulation

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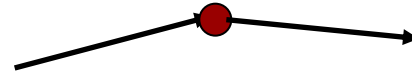
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  - Assume velocity is constant (or the acceleration is)
  - Compute the position
  - Move for next frame
- Movement is very linear
  - Piecewise approximations
  - Remember you calculus
- Smooth = smaller steps
  - More frames a second?



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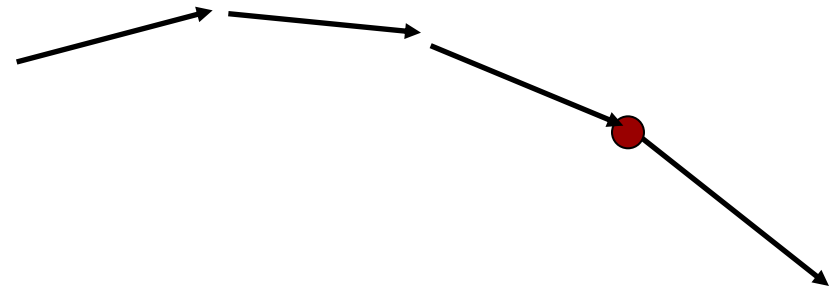
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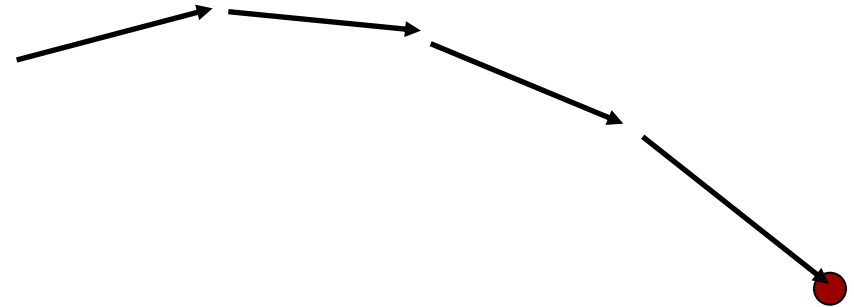
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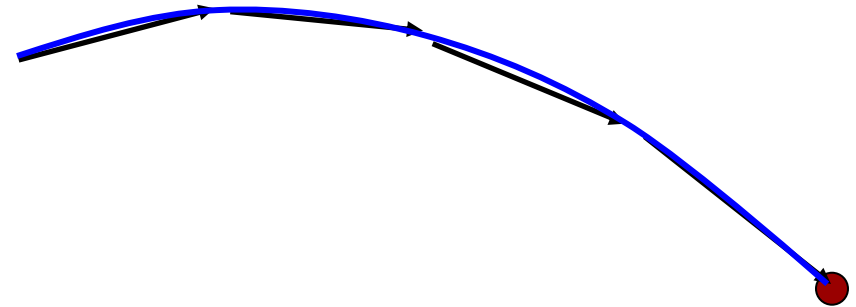
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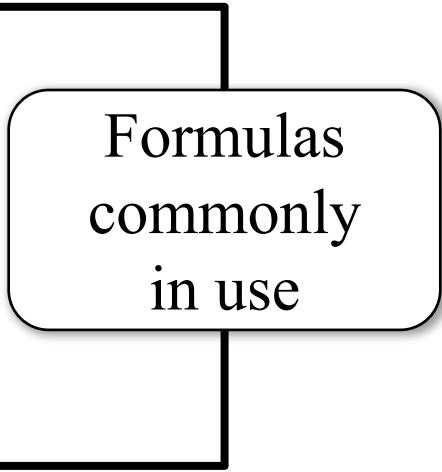
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# Kinematics

- **Goal:** determine an object position  $p$  at time  $t$ 
  - Typically know it from a previous time
- **Assume:** constant velocity  $v$ 
  - $p(t+\Delta t) = p(t) + v\Delta t$
  - Or  $\Delta p = p(t+\Delta t) - p(t) = v\Delta t$
- **Alternatively:** constant acceleration  $a$ 
  - $v(t+\Delta t) = v(t) + a\Delta t$  (or  $\Delta v = a\Delta t$ )
  - $p(t+\Delta t) = p(t) + v(t)\Delta t + \frac{1}{2}a(\Delta t)^2$
  - Or  $\Delta p = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$

Formulas  
commonly  
in use



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- **Alternate:** constant acceleration  $a$

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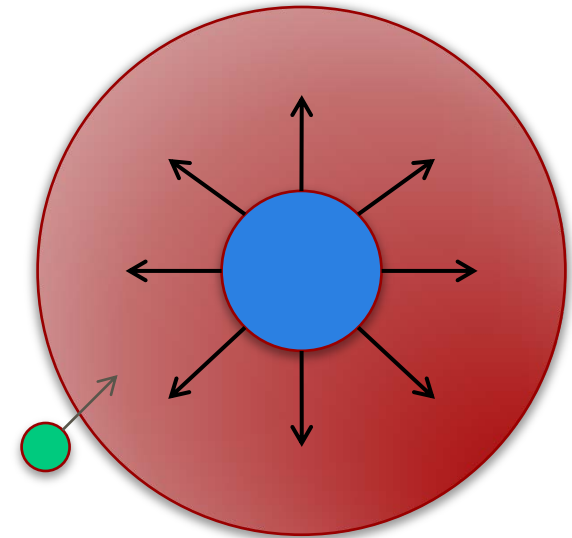
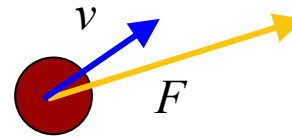
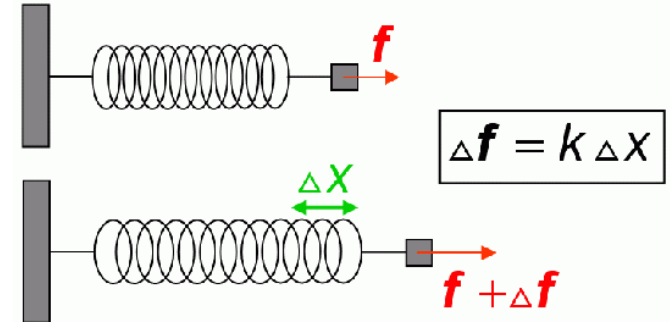
High School Physics w/o Calculus

Formulas  
commonly  
in use



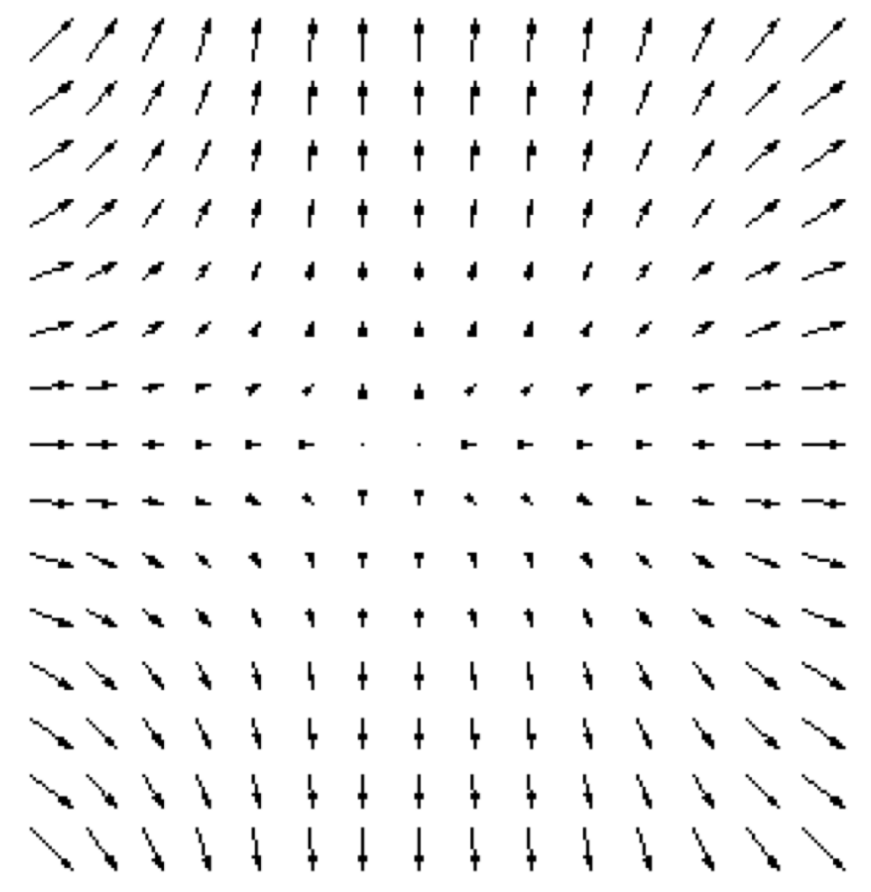
# Linear Dynamics

- **Forces** affect movement
  - Springs, joints, connections
  - Gravity, repulsion
- Get velocity from forces
  - Compute current force  $F$
  - **$F$  constant entire frame**
  - Formulas:
    - $\Delta a = F/m$
    - $\Delta v = F\Delta t/m$
    - $\Delta p = F(\Delta t)^2/m$
- Again, piecewise **linear**



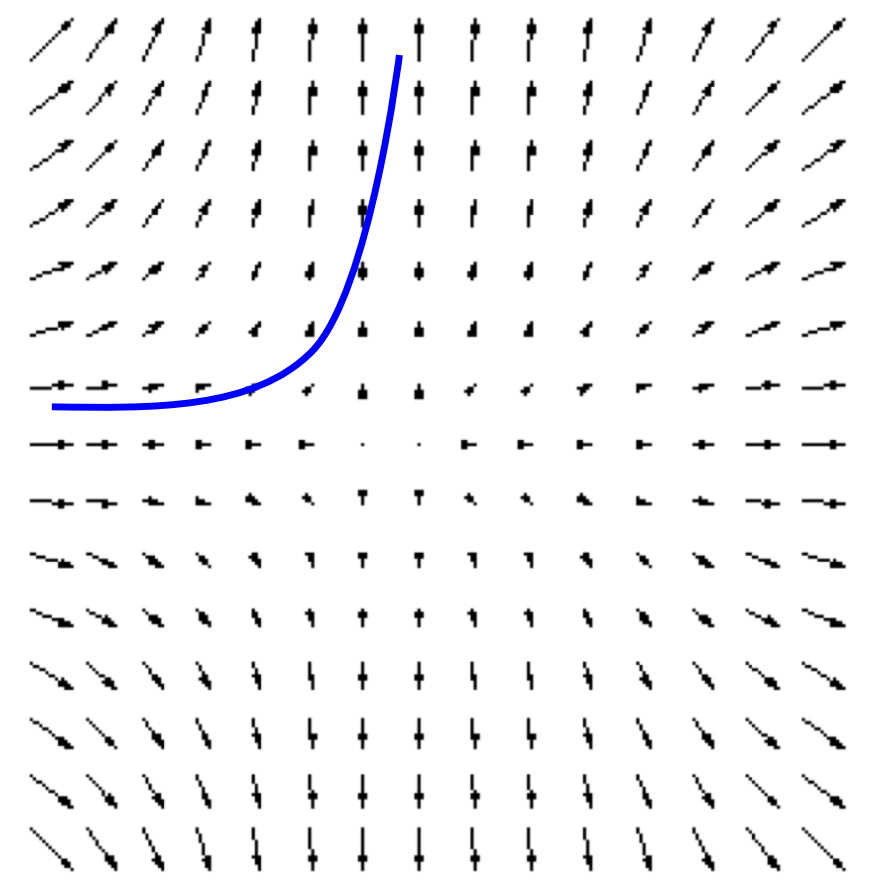
# Linear Dynamics

- **Force:**  $F(p,t)$ 
  - $p$ : current position
  - $t$ : current time
- Creates a **vector field**
  - Movement should follow field direction
- Update formulas
  - $a_i = F(p_i, i\Delta t)/m$
  - $v_{i+1} = v_i + a_i\Delta t$
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# Physics as DE Solvers

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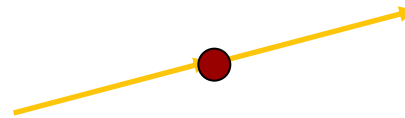
- Differential Equation
  - $F(p,t) = m a(t)$
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- Euler's method:
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  - $v_{i+1} = v_i + a_i\Delta t$
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- Other techniques exist
  - **Example:** Runga-Kutta



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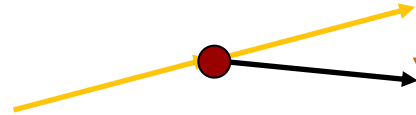
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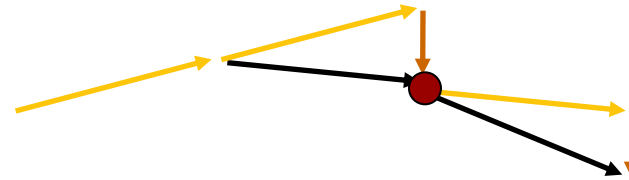
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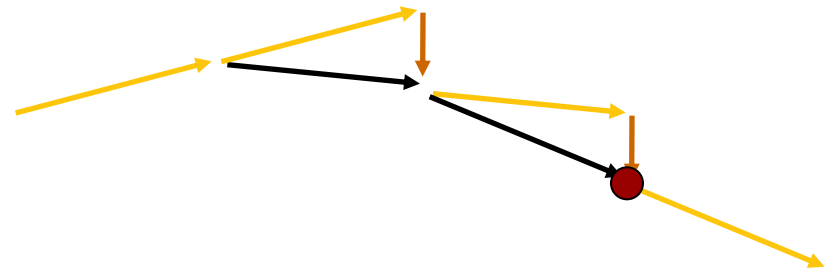
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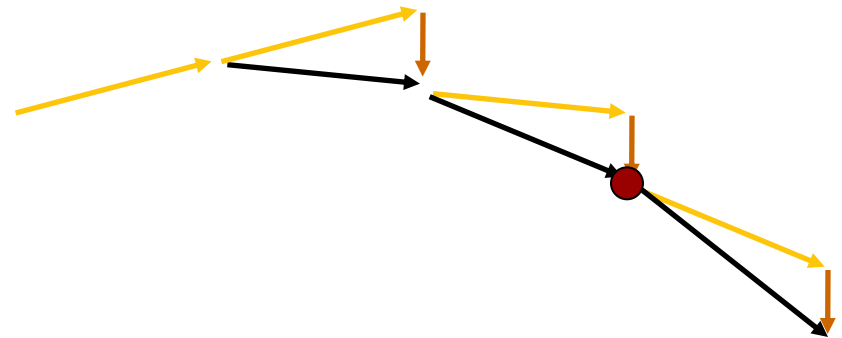
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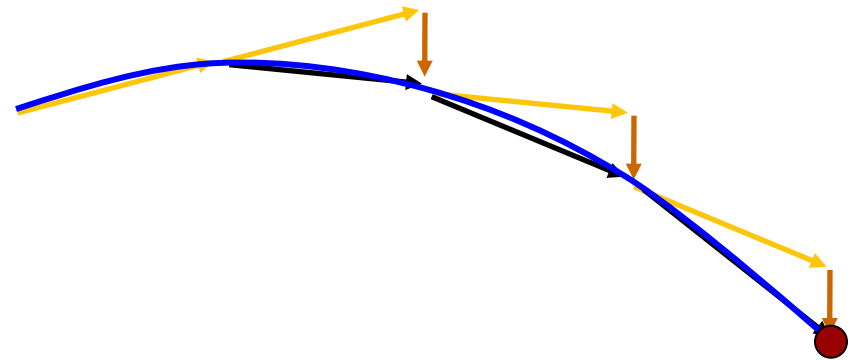
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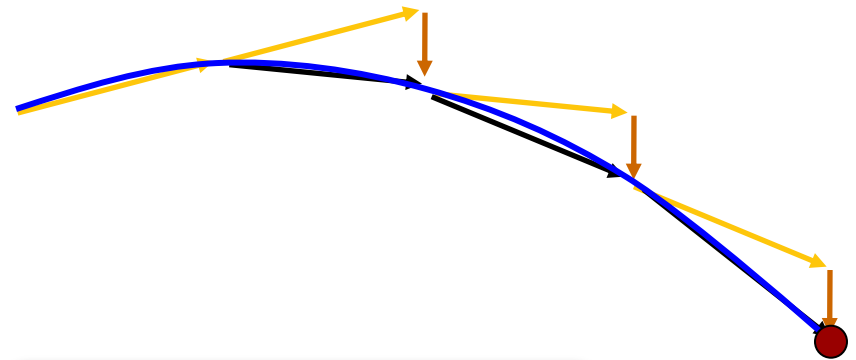
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Made for accuracy  
Not for speed

# Kinematics vs. Dynamics

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## Kinematics

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- **Advantages**

- Very simple to use
- Non-calculus physics

- **Disadvantages**

- Only simple physics
- All bodies are rigid

- Old school games

## Dynamics

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- **Advantages**

- Complex physics
- Non-rigid bodies

- **Disadvantages**

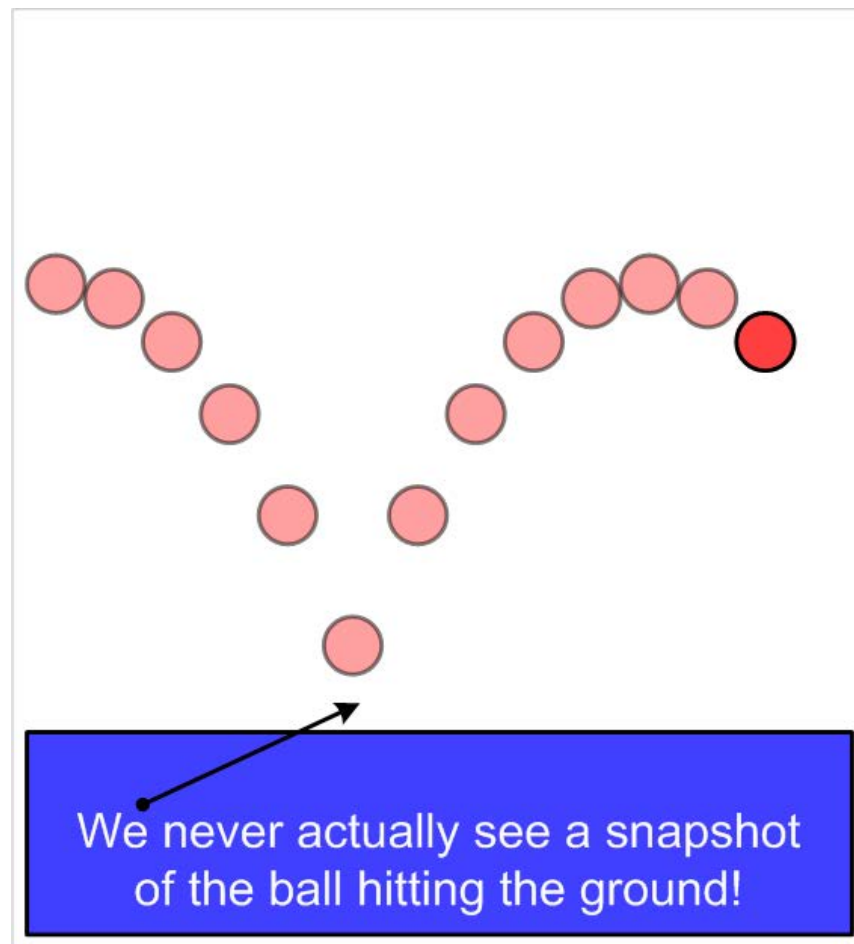
- Beyond scope of course
- Need a physics engine

- Neo-retro games

# Issues with Game Physics

## Flipbook Syndrome

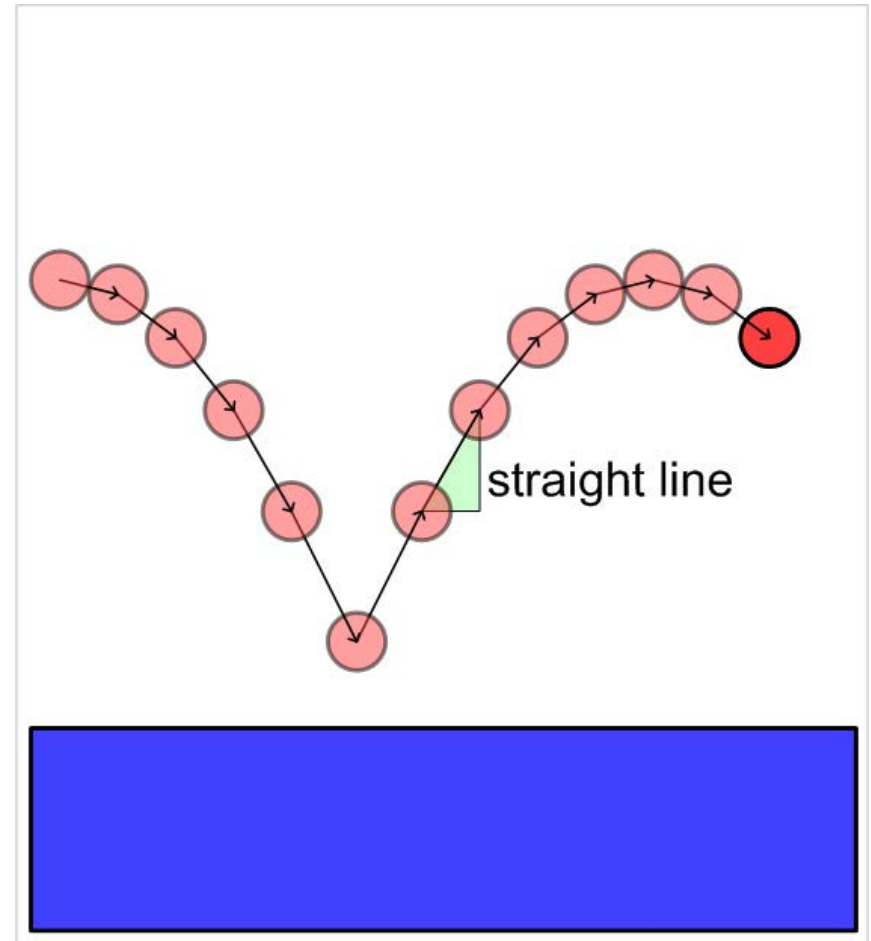
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- Curved trajectories are actually piecewise linear
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- Errors accumulate



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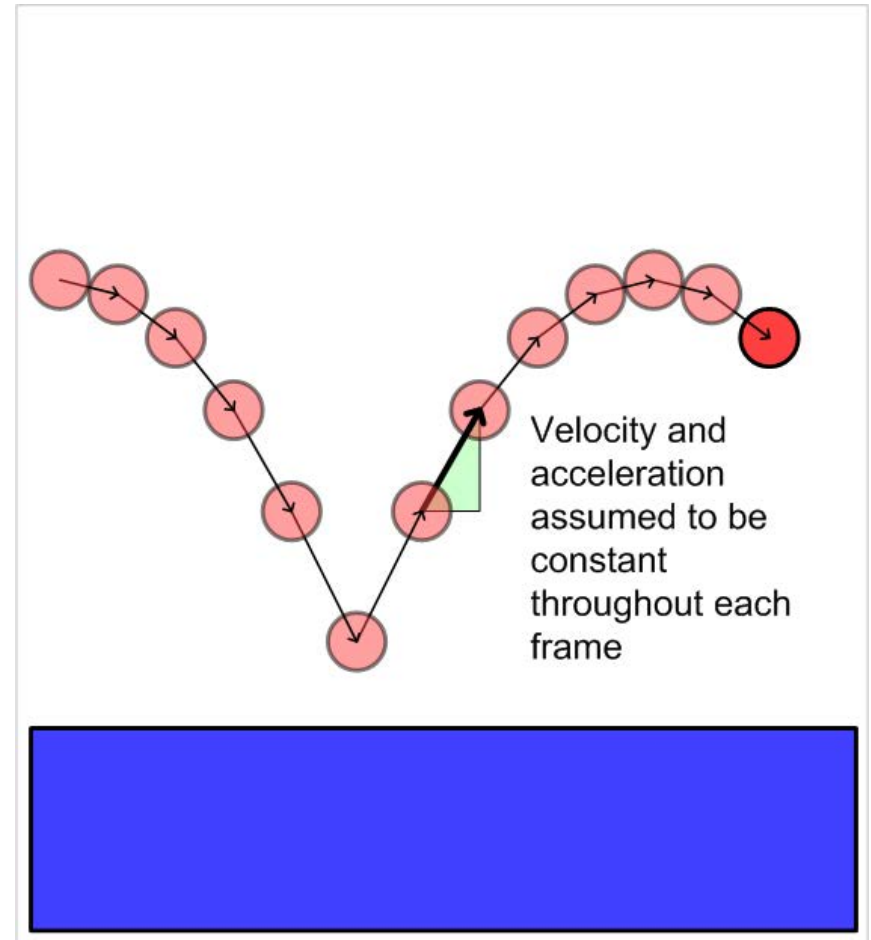
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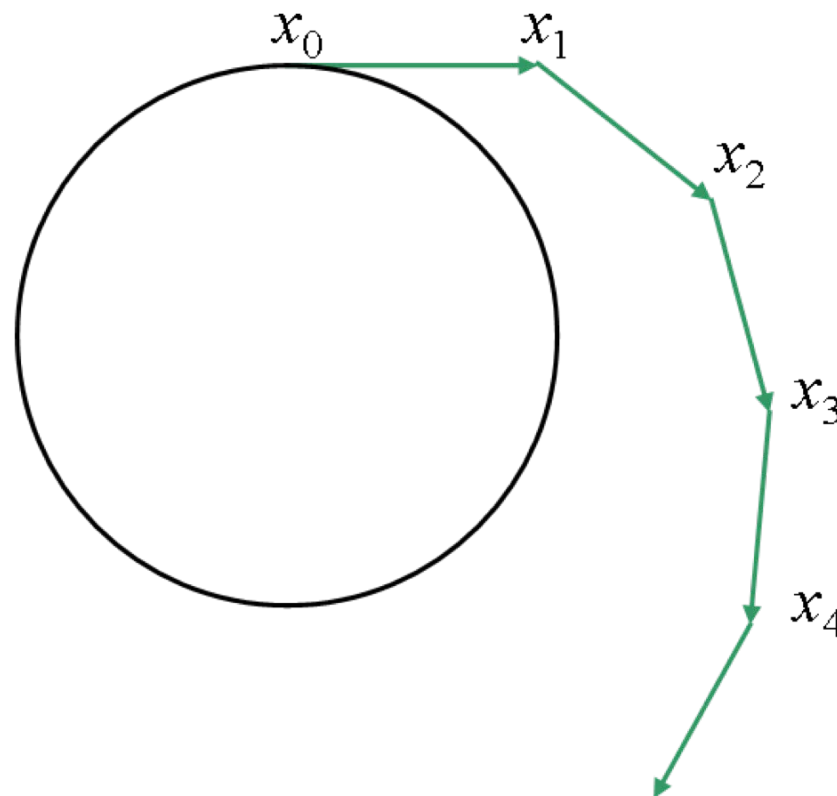




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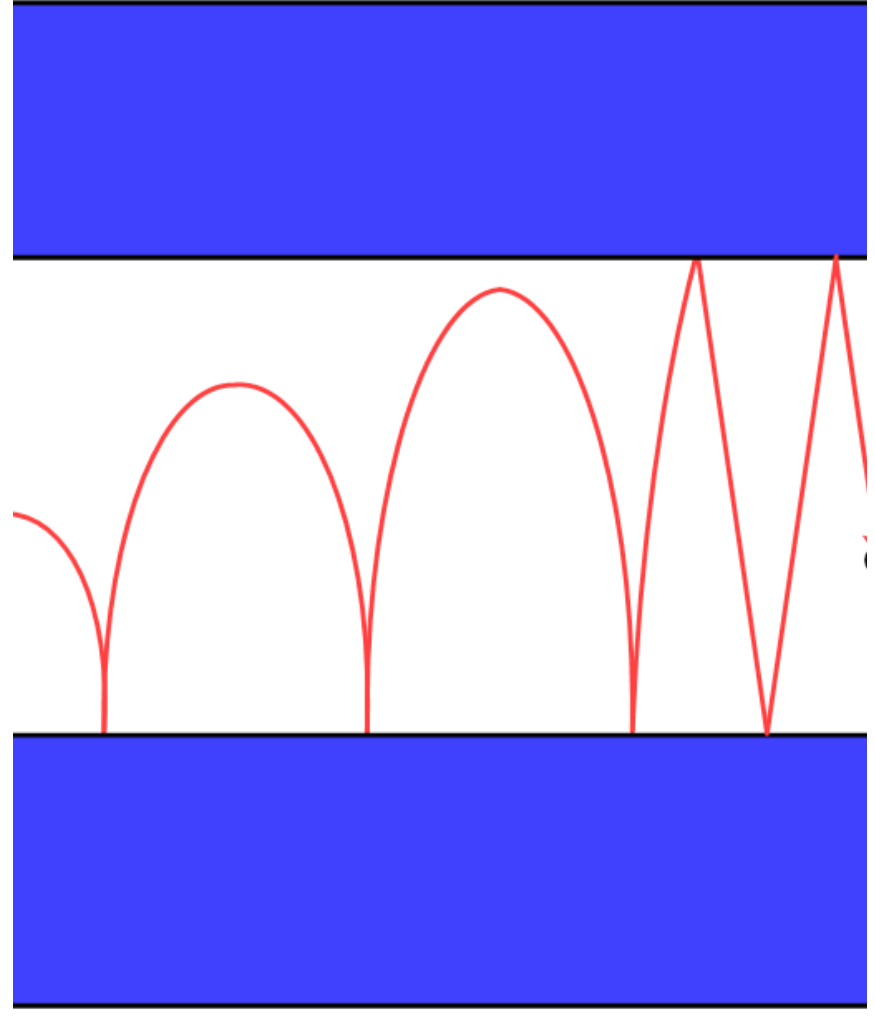
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# Issues with Game Physics

- Want energy conserved
  - Energy loss undesirable
  - Energy gain is **evil**
  - Simulations explode!
- Not always possible
  - Error accumulation
  - Visible artifact of Euler
- Requires **ad hoc** solutions
  - Clamping (max values)
  - Manual *dampening*



# Dealing with Error Creep

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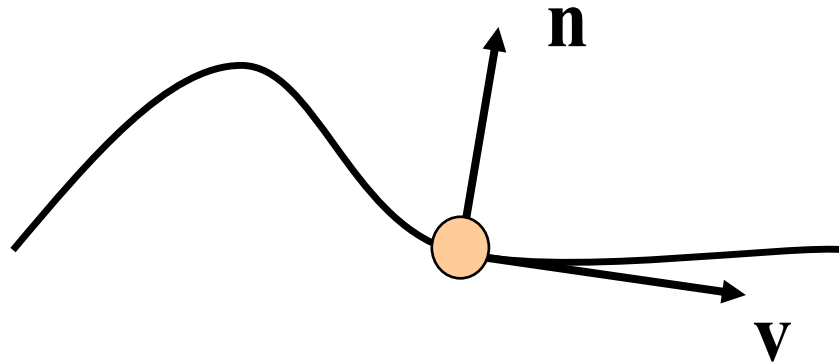
- Classic solution: reduce the time step  $\Delta t$ 
  - Up the frame rate (not necessarily good)
  - Perform more than one step per frame
  - Each Euler step is called an *iteration*
- **Multiple iterations per frame**
  - Let  $h$  be the length of the frame
  - Let  $n$  be the number of iterations
- Typically a parameter in your physics engine

$$\Delta t = h/n$$

# Constrained Particle Behavior

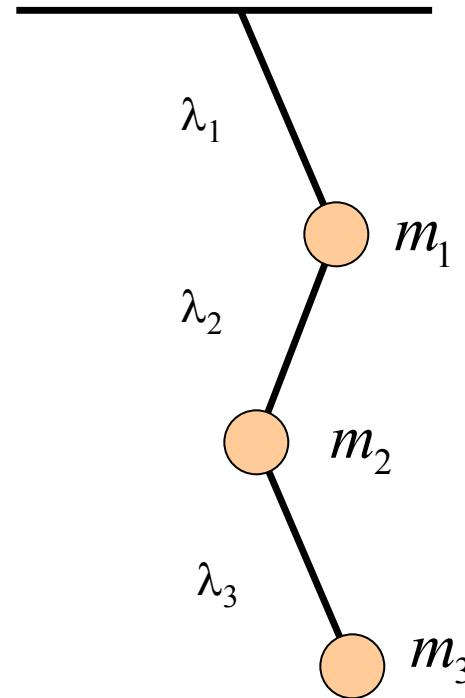
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- Suppose we have a bead on a wire
  - The bead can slide freely along wire
  - It can never come off, however hard we pull.
  - How does the bead move under applied forces?
- Usually a curve given by function  $C(x,y) = 0$



# Constraint Solvers

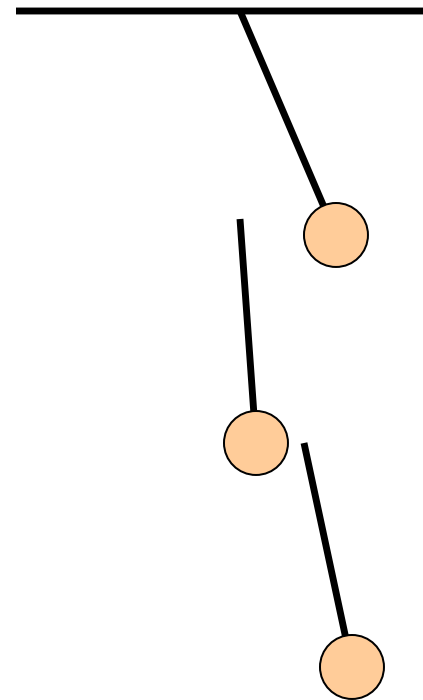
- **Limit** object movement
  - **Joints**: distance constraint
  - **Contact**: non-penetration
  - **Restitution**: bouncing
  - **Friction**: sliding, sticking
- Many applications
  - Ropes, chains
  - Box stacking
- Focus of Lab 4 (Box2D)



# Implementing Constraints

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- Very difficult to implement
  - **Errors:** joints to fall apart
  - Called *position drift*
  - Too hard for this course
- Use a physics engine!
  - Box2D supports constraints
  - Limit applications to joints
  - **Example:** ropes, rag dolls
- Want more? CS 5643
  - Or read about it online



# Physics in Games

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(Only consider position, velocity, acceleration)
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# Collisions and Geometry

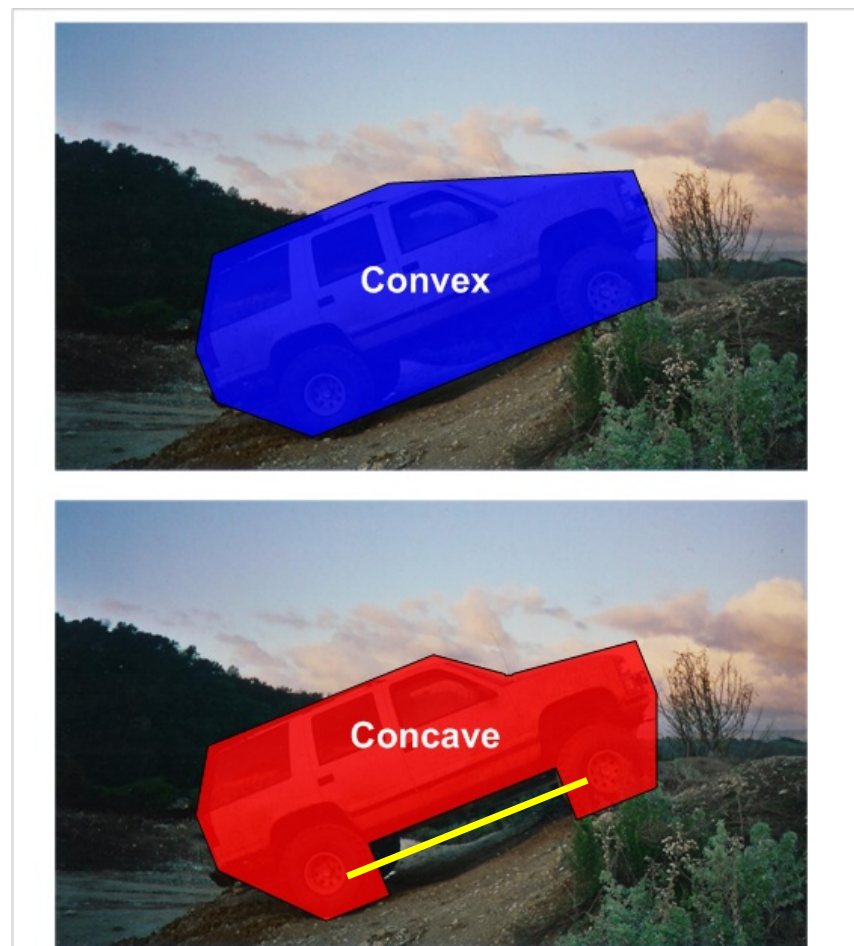
- Collisions require **geometry**
  - Points are no longer enough
  - Must know *where* objects meet
- Often use convex shapes
  - Lines always remain inside
  - If not convex, call it concave
  - Easiest shapes to compute with
- What to do if is not convex?
  - Break into convex components
  - Triangles are always convex!





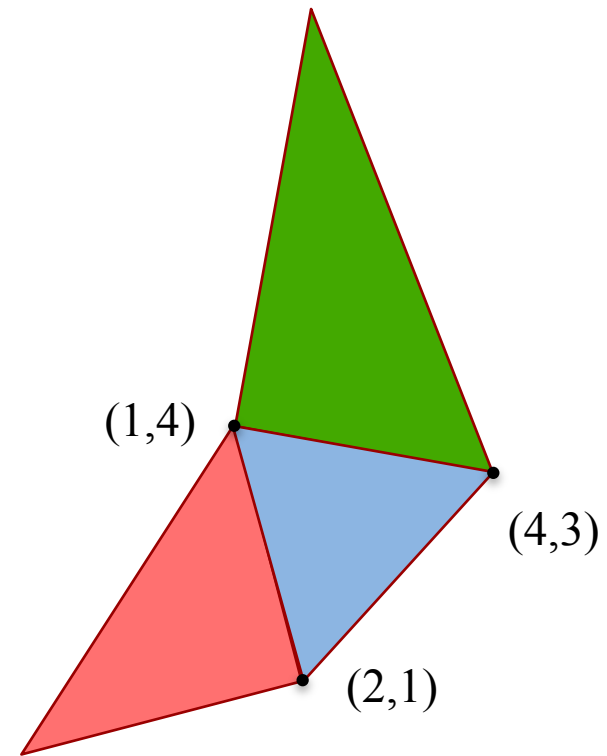
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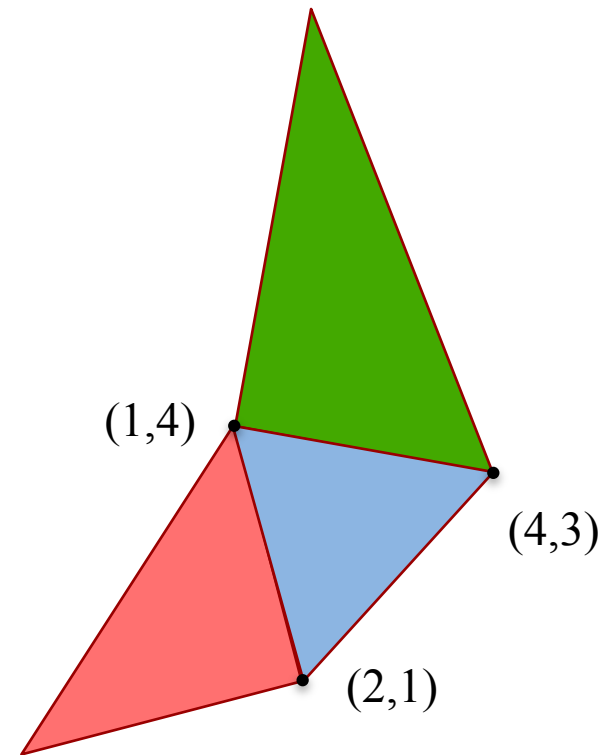
# Recall: Triangles in Computer Graphics

- Everything made of **triangles**
  - Mathematically “nice”
  - Hardware support (GPUs)
- Specify with **three vertices**
  - Coordinates of corners
- Composite for complex shapes
  - Array of vertex objects
  - Each 3 vertices = triangle



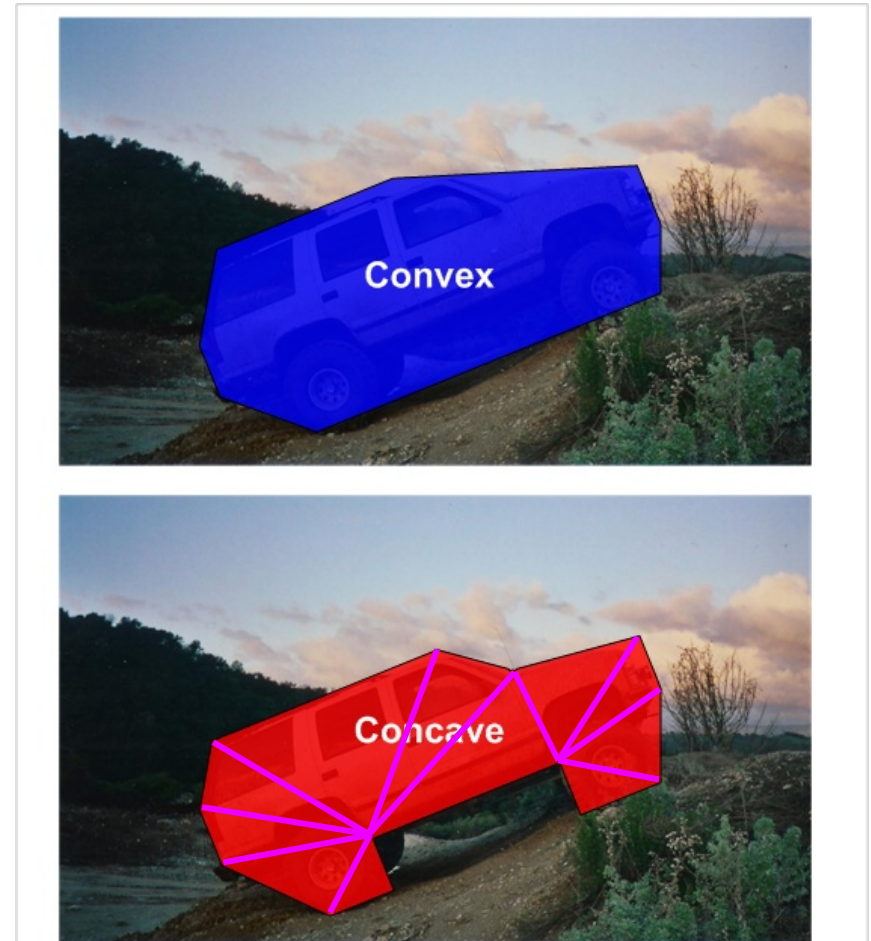
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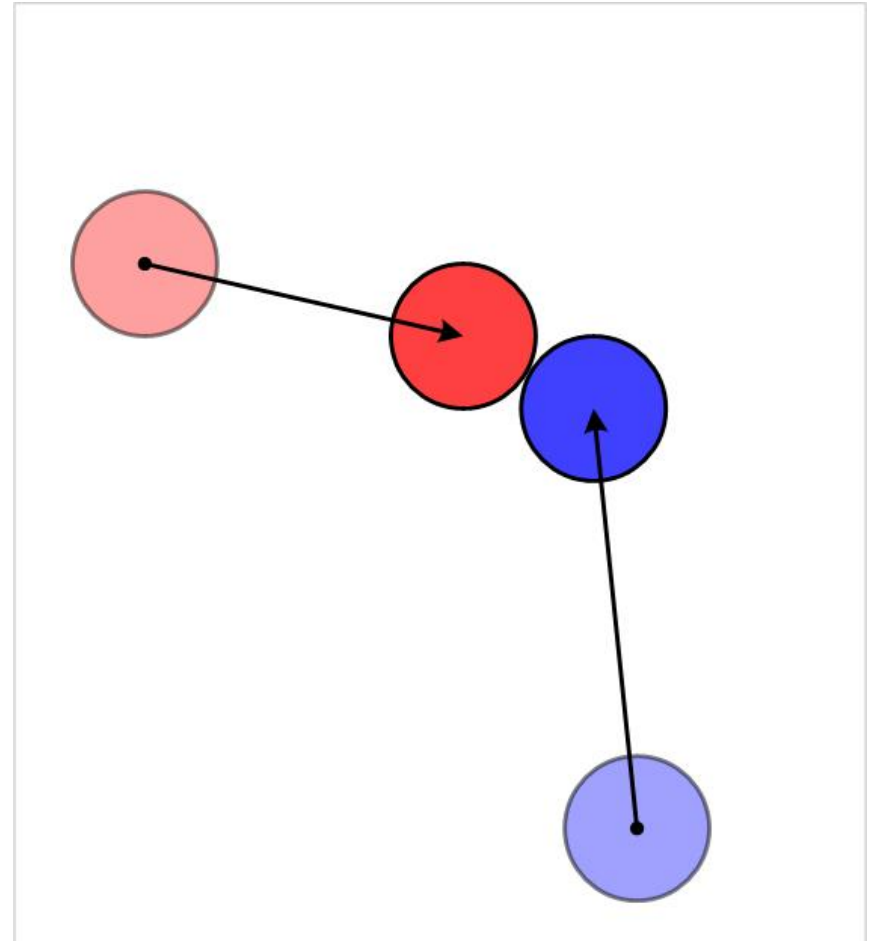
# Collision Types

- **Inelastic Collisions**

- No energy preserved
- Stop in place ( $v = 0$ )
- “Back-out” so no overlap
- Very easy to implement

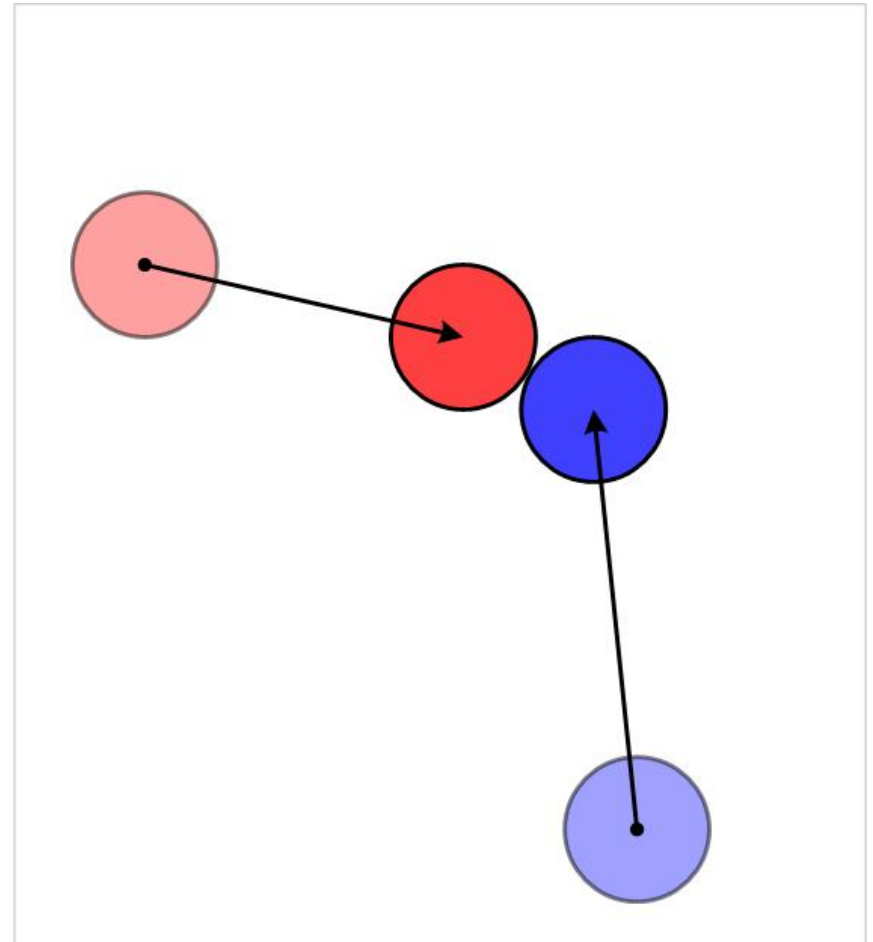
- **Elastic Collisions**

- 100% energy preserved
- Think billiard balls
- Classic physics problem



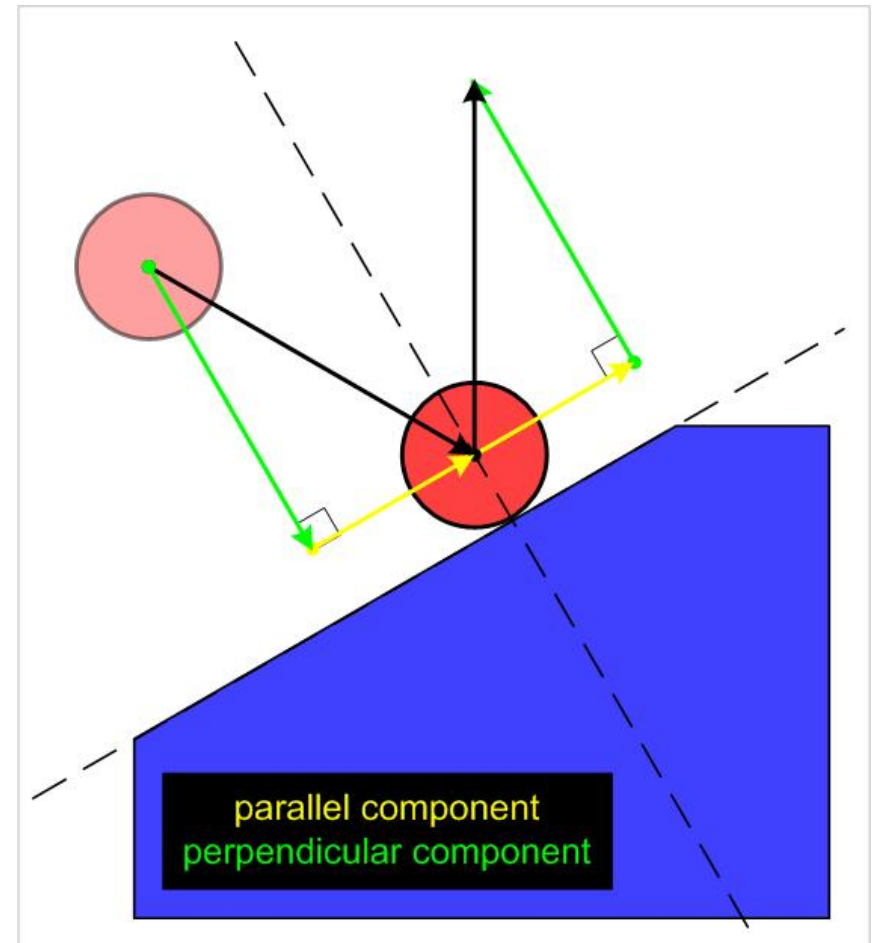
# Something In-Between?

- **Partially Elastic**
  - $x\%$  energy preserved
  - Different each object
  - Like elastic, but harder
- **Issue:** object “material”
  - What is object made of?
  - **Example:** Rubber? Steel?
- Another parameter!
  - Technical prototype?



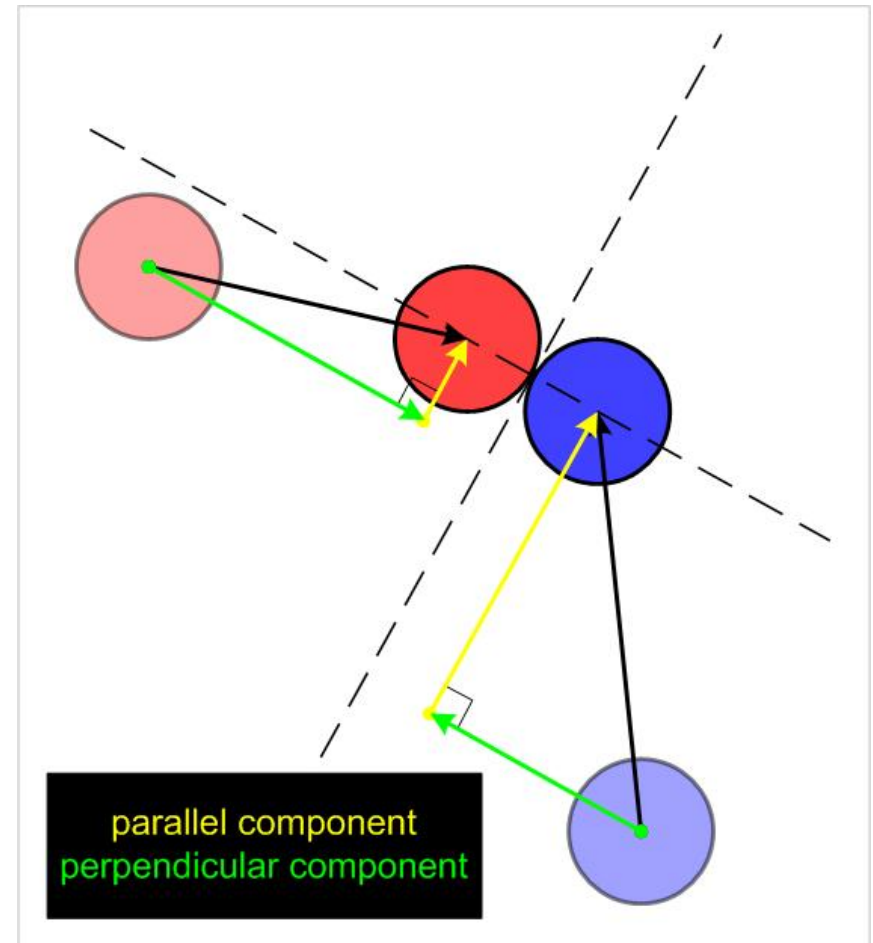
# Collision Resolution: Circles

- Single point of contact!
  - Energy transferred at point
  - Not true in complex shapes
- Use **relative coordinates**
  - Point of contact is origin
  - **Perpendicular component:**  
Line through origin, center
  - **Parallel component:**  
Axis of collision “surface”
- Reverse object motion on the perpendicular comp



# Collision Resolution: Circles

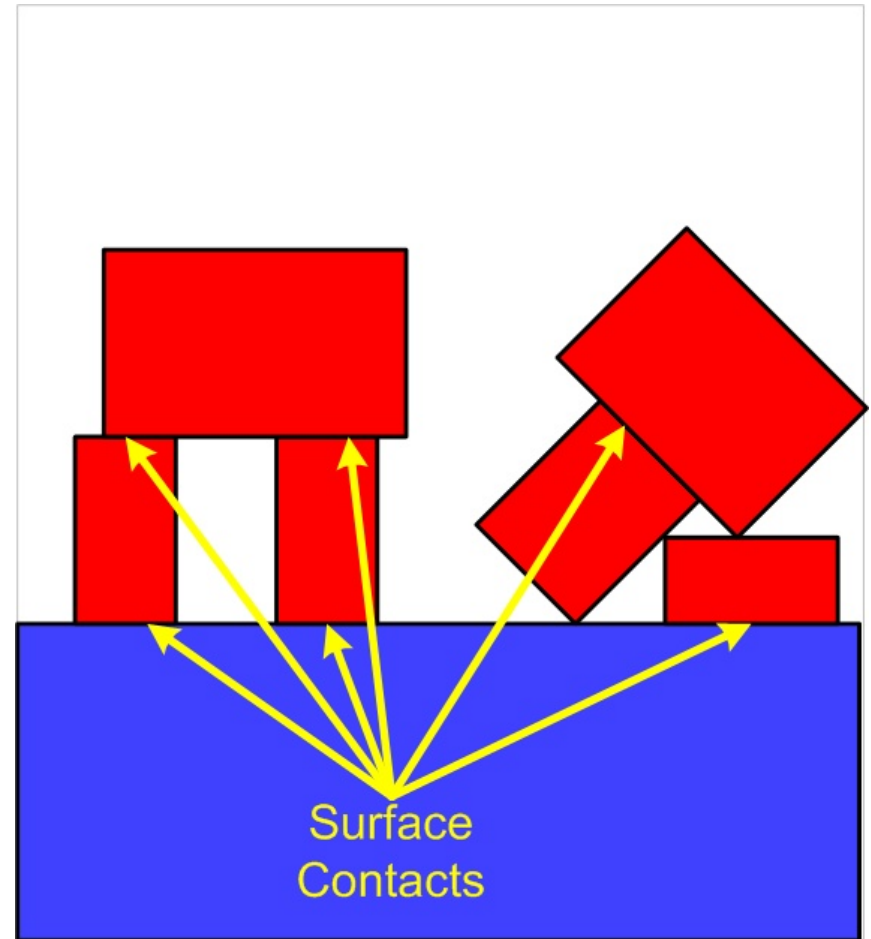
- Single point of contact!
  - Energy transferred at point
  - Not true in complex shapes
- Use **relative coordinates**
  - Point of contact is origin
  - **Perpendicular component:**  
Line through origin, center
  - **Parallel component:**  
Axis of collision “surface”
- **Exchange energy** on the perpendicular comp





# More Complex Shapes

- Point of contact harder
  - Could just be a point
  - Or it could be an edge
- Model with **rigid bodies**
  - Break object into points
  - Connect with constraints
  - Force at point of contact
  - Transfers to other points
- Needs **constraint solver**



# Summary

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- Object representation depends on goals
  - For **motion**, represent object as a **single point**
  - For **collision**, objects must have **geometry**
- Dynamics is the use of forces to move objects
  - **Particle systems**: objects exert a force on one another
  - **Constraint solvers**: restrictions for more rigid behavior
- Collisions are broken up into two steps
  - **Collision detection** checks for intersections
  - **Collision resolution** depends on energy transfer