the gamedesigninitiative at cornell university

Lecture 18

Physics: Overview

The Pedagogical Problem

- Physics simulation is a **very** complex topic
  - No way I can address this in a few lectures
  - Could spend an entire course talking about it
  - **CS 5643**: Physically Based Animation
- This is why we have **physics engines**
  - Libraries that handle most of the dirty work
  - But you have to understand how they work
  - **Examples**: Box2D (Farseer), Chipmunk, Bullet

Approaching the Problem

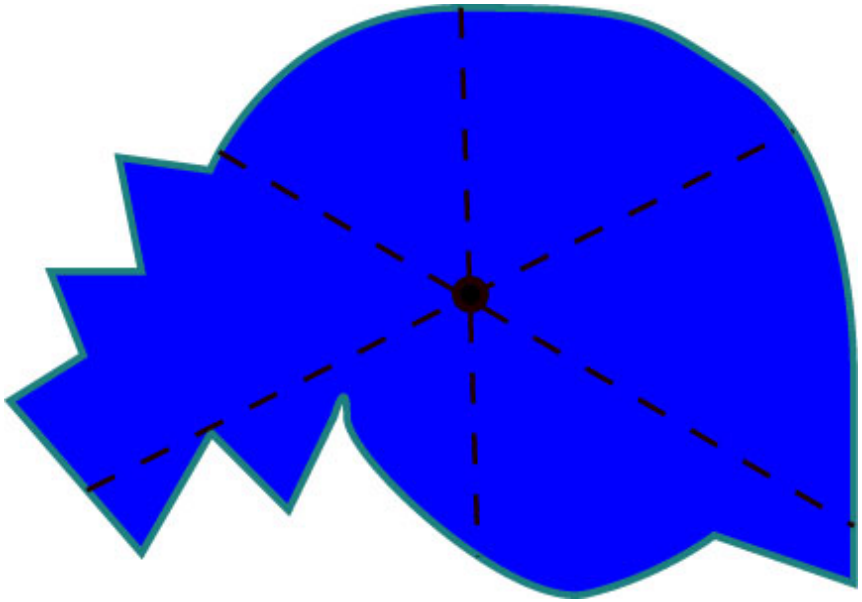
- Want to start with the **problem description**
  - Squirrel Eiserloh's *Problem Overview* slides
  - http://www.essentialmath.com/tutorial.htm
- Will help you understand the Engine APIs
  - Understand the limitations of physics engines
  - Learn where to go for other solutions
- Will cover Box2D API next time in depth

Physics in Games

- **Moving** objects about the screen
  - **Kinematics**: Motion ignoring external forces (Only consider position, velocity, acceleration)
  - **Dynamics**: The effect of forces on the screen
- **Collisions** between objects
  - **Collision Detection**: Did a collision occur?
  - **Collision Resolution**: What do we do?

Motion: Modeling Objects

- Typically ignore **geometry**
  - Don't worry about shape
  - Only needed for *collisions*
- Every object is a **point**
  - *Centroid*: average of points
  - Also called: *center of mass*
  - Same if density uniform
- Use **rigid body** if needed
  - Multiple points together
  - Moving one moves them all



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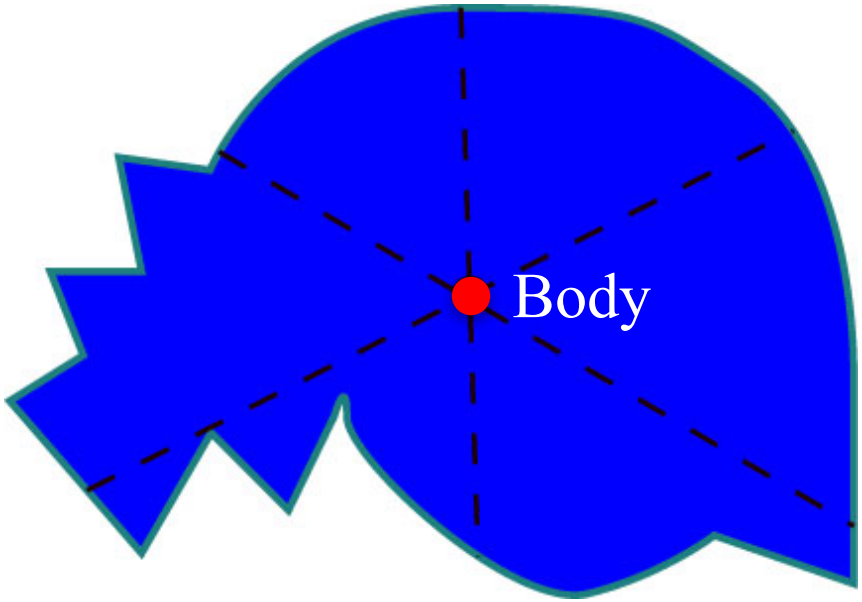

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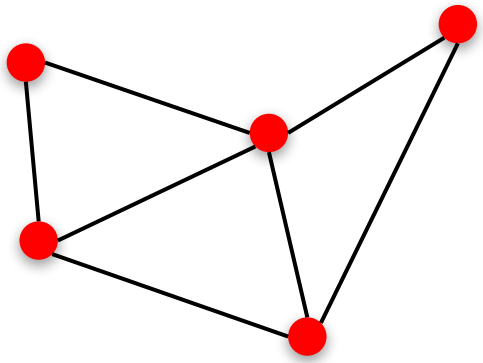
Rigid Body



Time-Stepped Simulation

- Physics is **time-stepped**
  - Assume velocity is constant (or the acceleration is)
  - Compute the position
  - Move for next frame

- Movement is very linear
  - Piecewise approximations
  - Remember you calculus
- Smooth = smaller steps
  - More frames a second?

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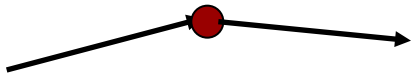

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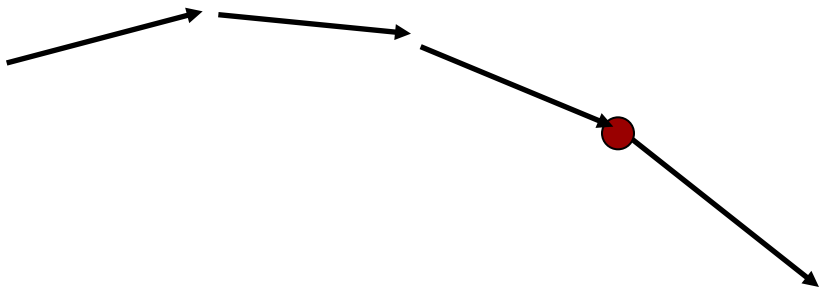

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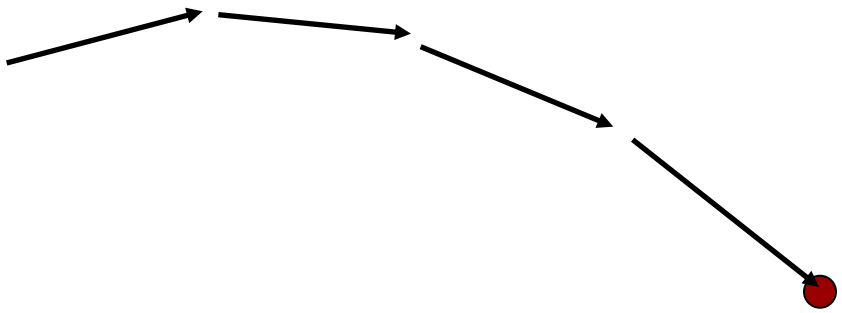

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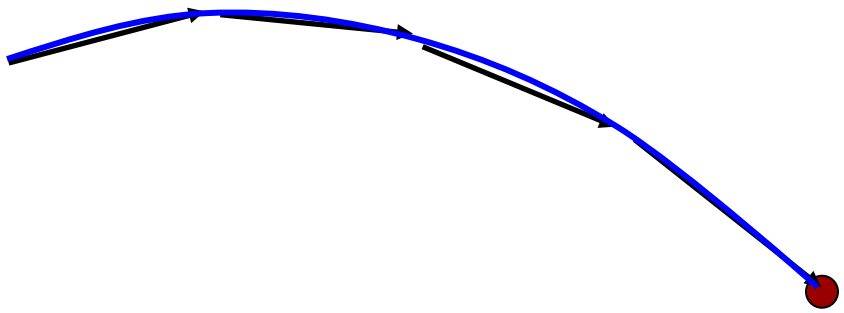

Kinematics

- **Goal**: determine an object position p at time t
  - Typically know it from a previous time
- **Assume**: constant velocity v
  - $p(t+\Delta t) = p(t) + v\Delta t$
  - Or $\Delta p = p(t+\Delta t)-p(t) = v\Delta t$
- **Alternatively**: constant acceleration a
  - $v(t+\Delta t) = v(t) + a\Delta t$ (or $\Delta v = a\Delta t$)
  - $p(t+\Delta t) = p(t) + v(t)\Delta t + \frac{1}{2}a(\Delta t)^2$
  - Or $\Delta p = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$

Formulas commonly in use

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- **Assume**: constant velocity v
  - $p(t+\Delta t) = p(t) + v\Delta t$
  - Or $\Delta p = p(t+\Delta t)$ [illegible]
- **Alt**[illegible] acceleration a
  - [illegible] $a\Delta t$ (or $\Delta v = a\Delta t$)
  - [illegible] $\Delta t) = p(t) + v(t)\Delta t + \frac{1}{2}a(\Delta t)^2$
  - Or $\Delta p = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$

High School Physics w/o Calculus

Formulas commonly in use

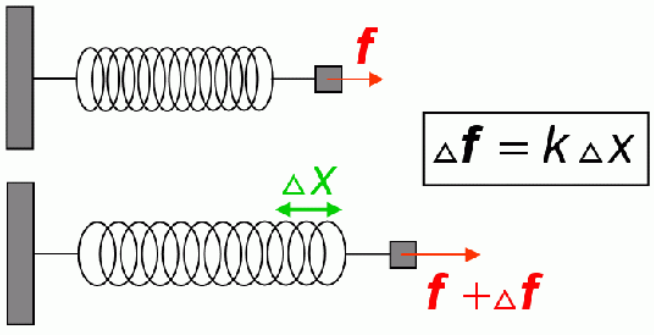
Linear Dynamics

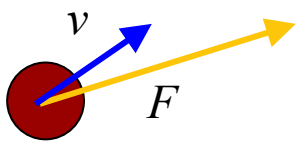
- **Forces** affect movement
  - Springs, joints, connections
  - Gravity, repulsion
- Get velocity from forces
  - Compute current force F
  - ***F* constant entire frame**
  - Formulas:

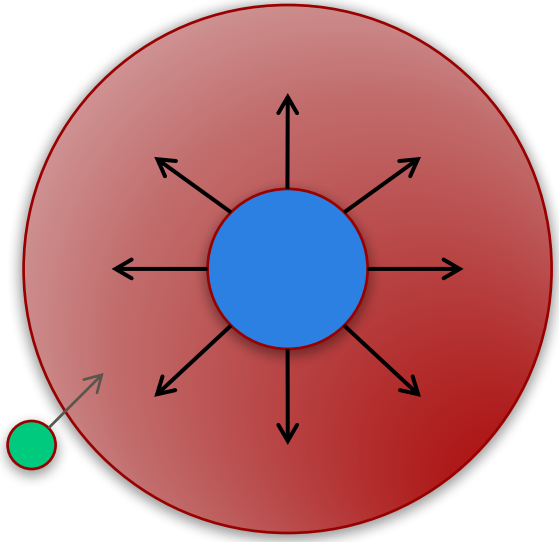
    $\Delta a = F/m$

    $\Delta v = F\Delta t/m$

    $\Delta p = F(\Delta t)^2/m$
- Again, piecewise **linear**







Linear Dynamics

- **Force**: $F(p,t)$
  - p: current position
  - t: current time
- Creates a **vector field**
  - Movement should follow field direction
- Update formulas
  - $a_i = F(p_i, i\Delta t)/m$
  - $v_{i+1} = v_i + a_i \Delta t$
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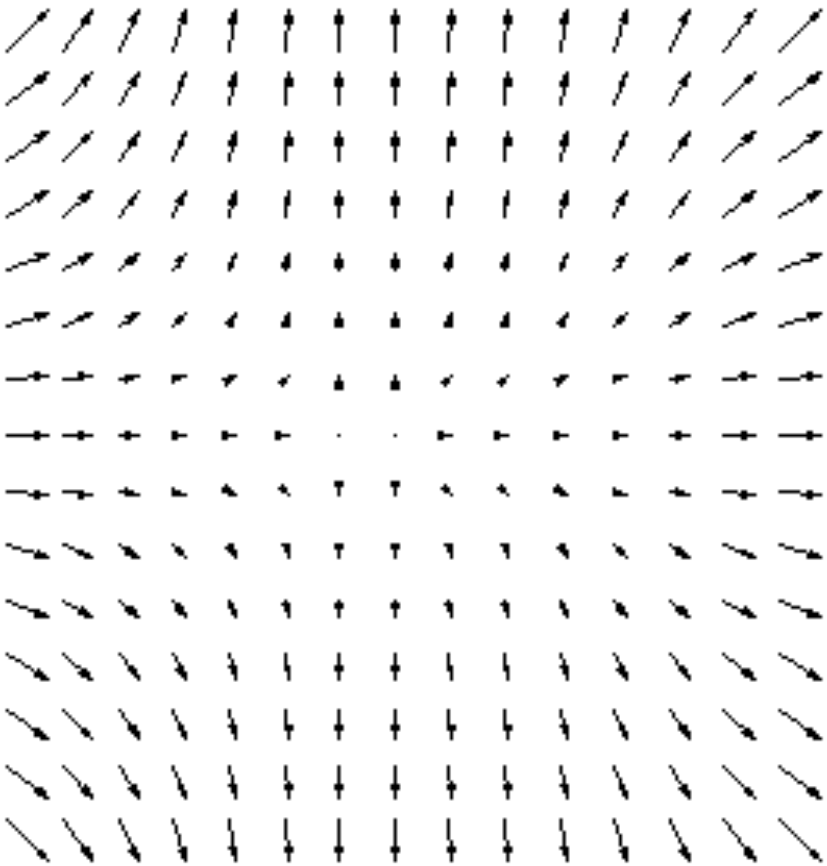

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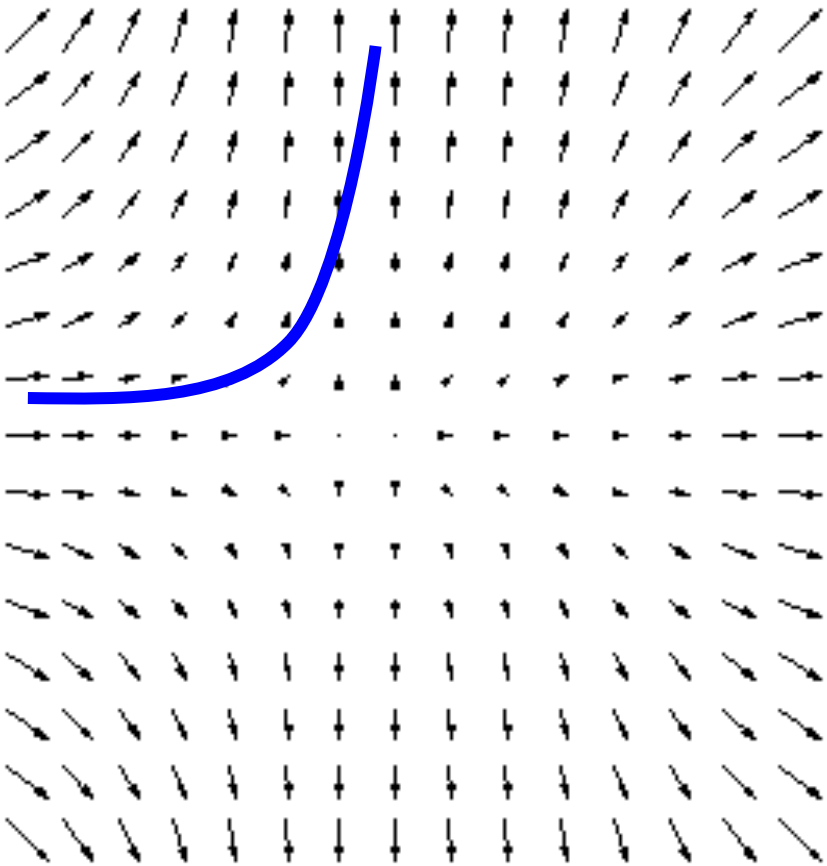

Physics as DE Solvers

- Differential Equation
  - $F(p,t) = m\ a(t)$
  - $F(p,t) = m\ \underline{p}''(t)$
- Euler's method:
  - $a_i = F(p_i, i\Delta t)/m$
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- Other techniques exist
  - **Example**: Runga-Kutta

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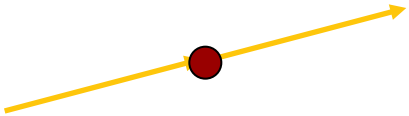

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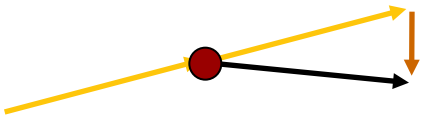

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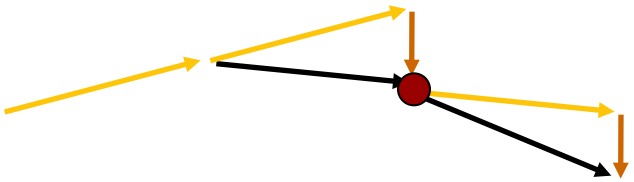

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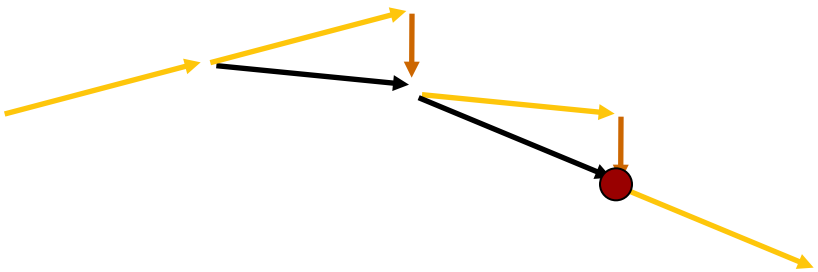

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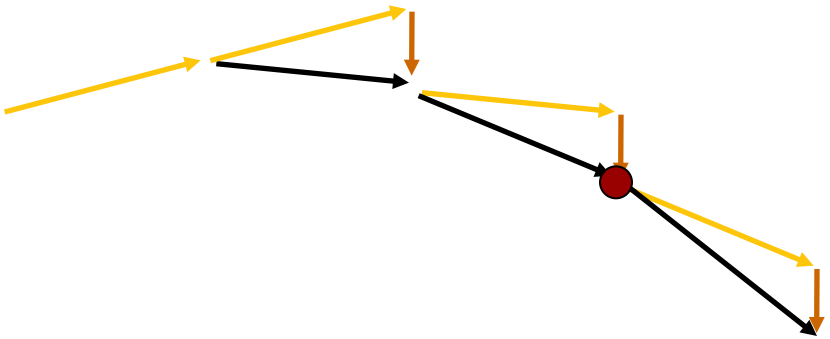

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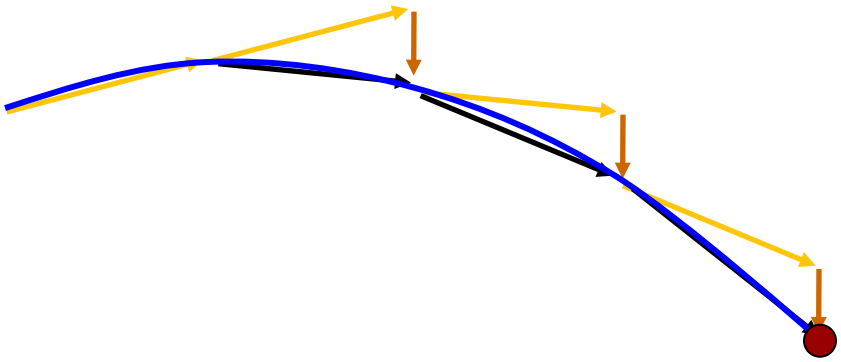

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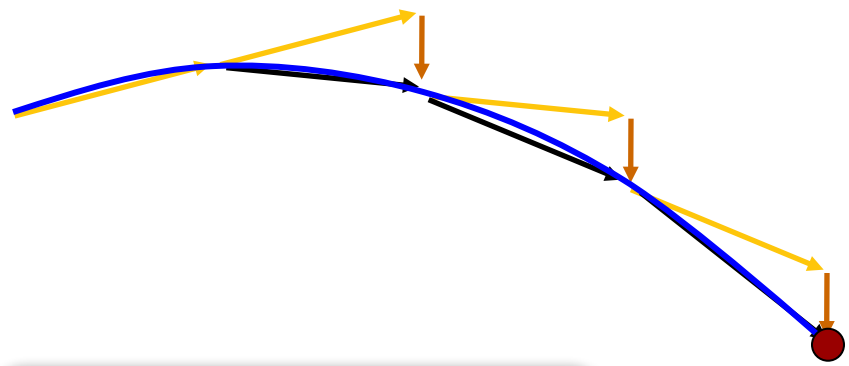


Made for accuracy
Not for speed

Kinematics vs. Dynamics

Kinematics

- **Advantages**
  - Very simple to use
  - Non-calculus physics
- **Disadvantages**
  - Only simple physics
  - All bodies are rigid
- Old school games

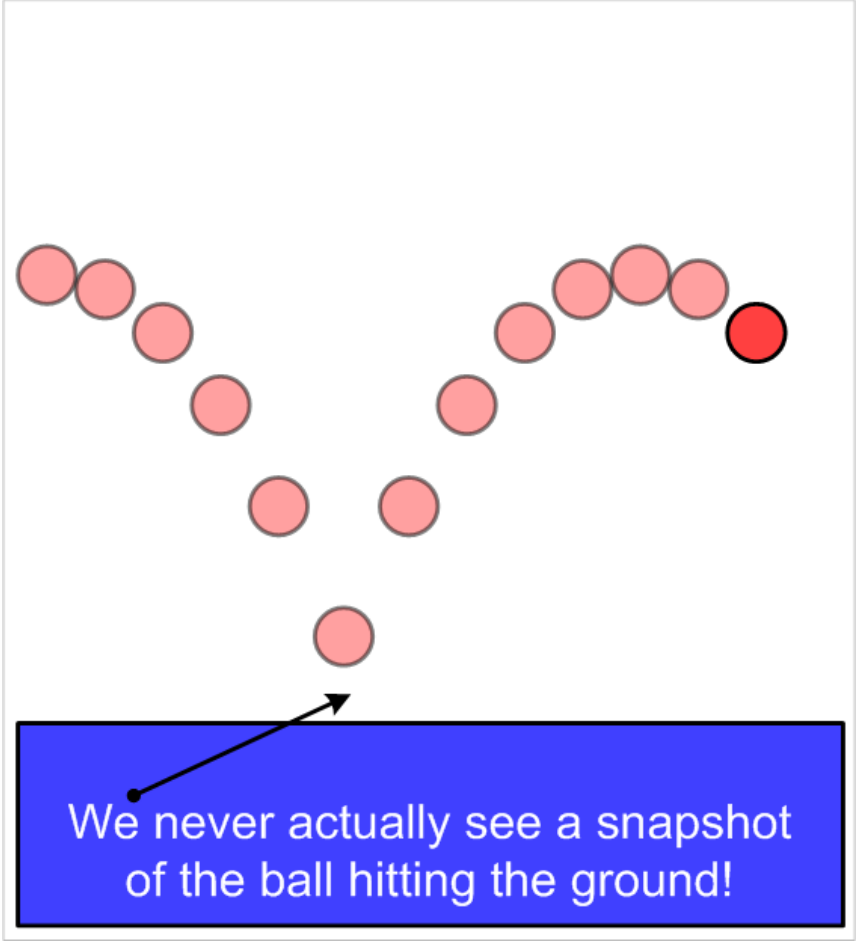
Dynamics

- **Advantages**
  - Complex physics
  - Non-rigid bodies
- **Disadvantages**
  - Beyond scope of course
  - Need a physics engine
- Neo-retro games

Issues with Game Physics

Flipbook Syndrome

- Things typically happen **in-between snapshots**
- Curved trajectories are actually piecewise linear
- Terms assumed constant throughout the frame
- Errors accumulate



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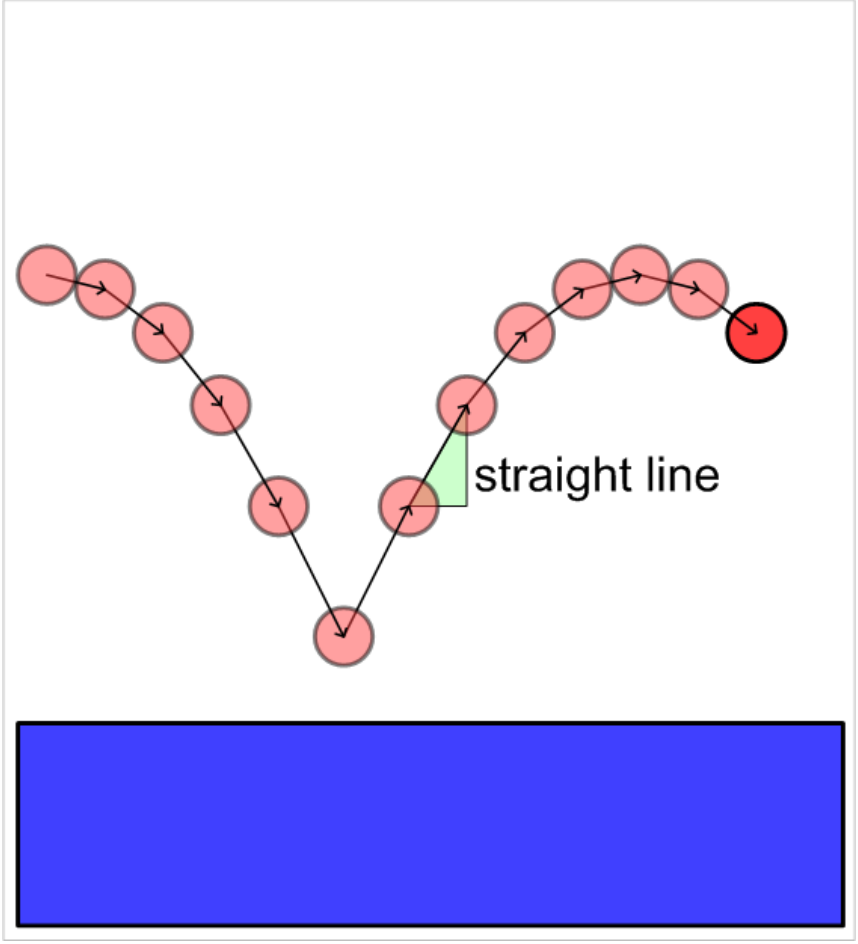

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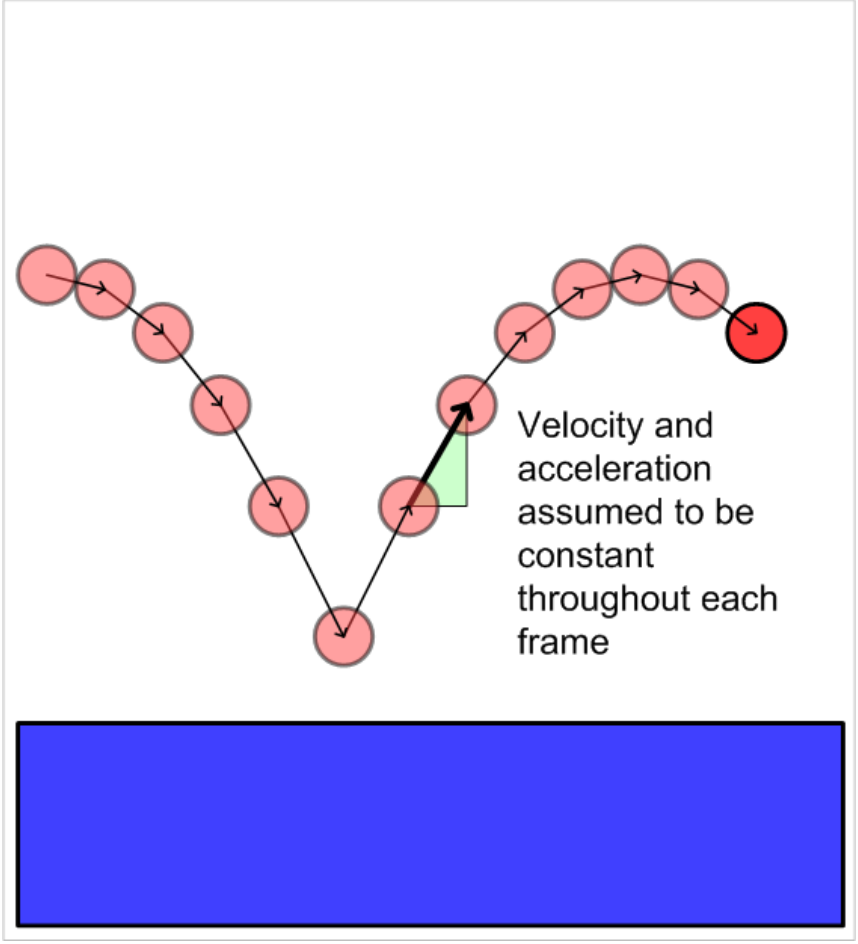

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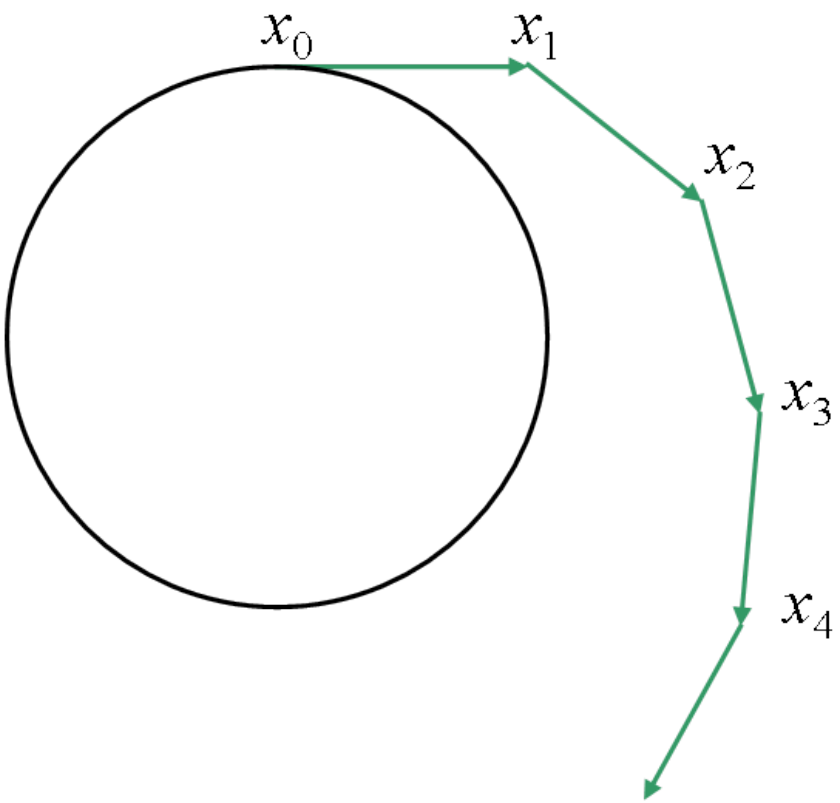
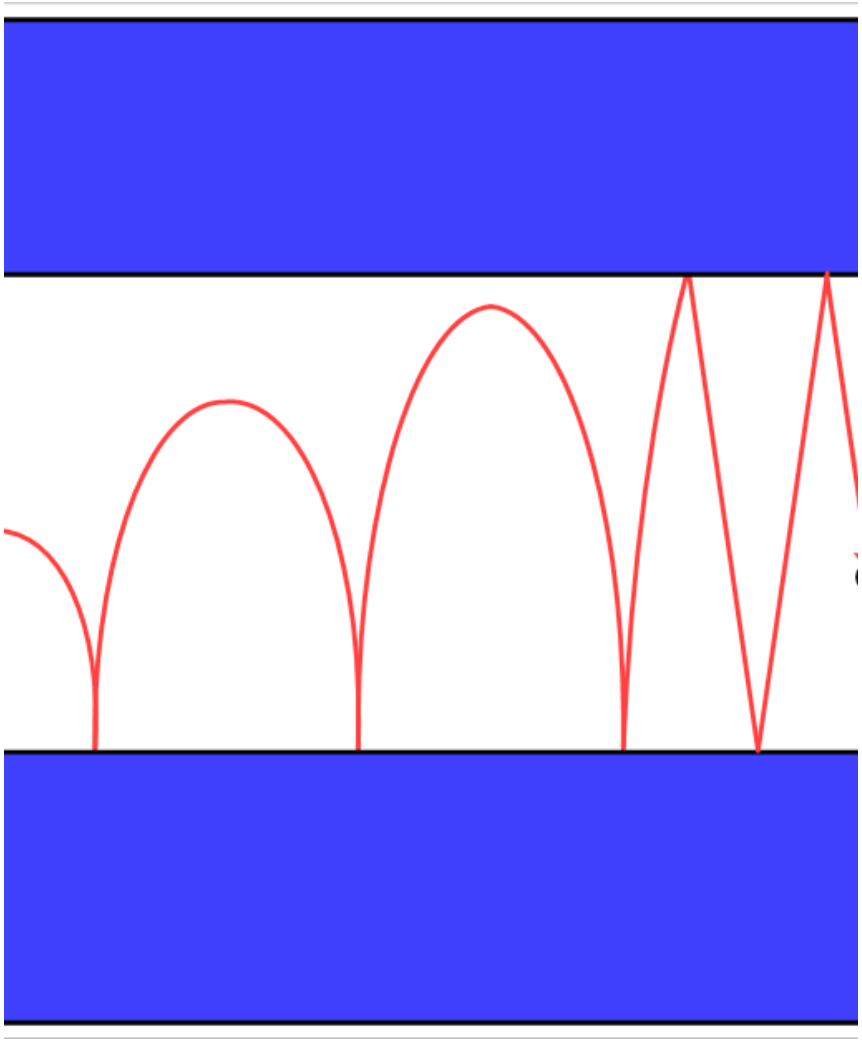


Issues with Game Physics

- Want energy conserved
  - Energy loss undesirable
  - Energy gain is **evil**
  - Simulations explode!
- Not always possible
  - Error accumulation
  - Visible artifact of Euler
- Requires **ad hoc** solutions
  - Clamping (max values)
  - Manual *dampening*



Dealing with Error Creep

- Classic solution: reduce the time step Δt
  - Up the frame rate (not necessarily good)
  - Perform more than one step per frame
  - Each Euler step is called an *iteration*
- **Multiple iterations per frame**
  - Let h be the length of the frame
  - Let n be the number of iterations

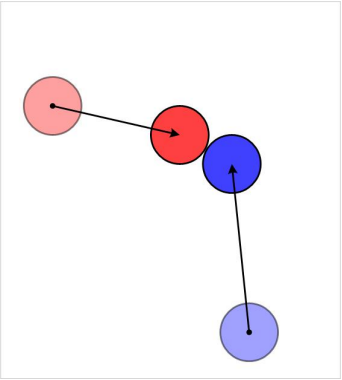
$$\Delta t = h/n$$

- Typically a parameter in your physics engine

Interactions of Objects

- **Collisions**
  - Typically assume elastic
  - 100% energy conserved
  - Think billiard balls
- **Springs**
  - Exerts a force on object
  - If too stretched, pulls back
  - If compressed, pushes out
  - Complex if ends not fixed
- Repulsive, **attractive** forces

Collisions



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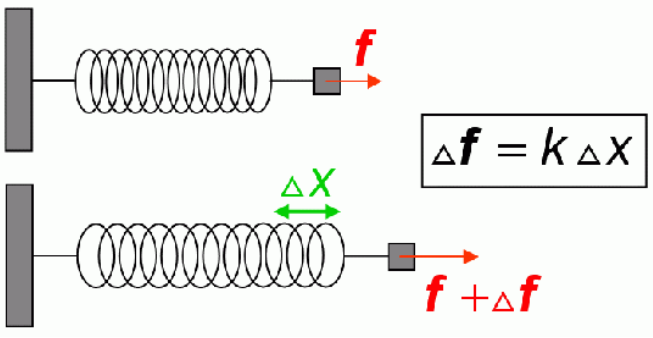

Interactions of Objects

- Particle Systems
- Billiard balls

- **Springs**
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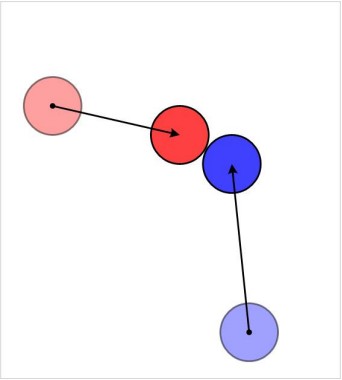

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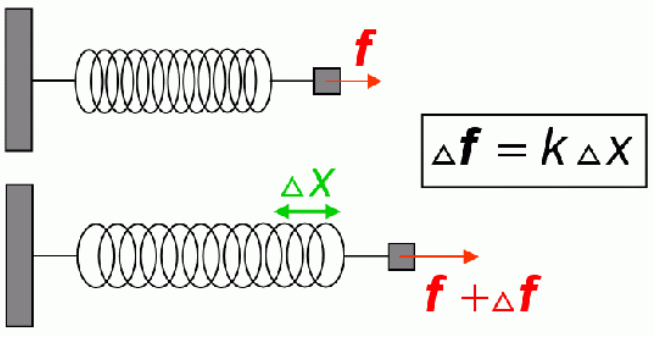
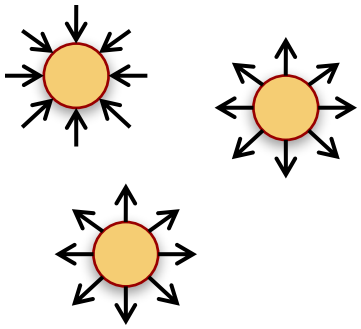
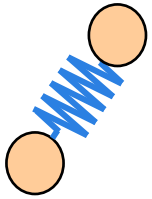


Particle Systems

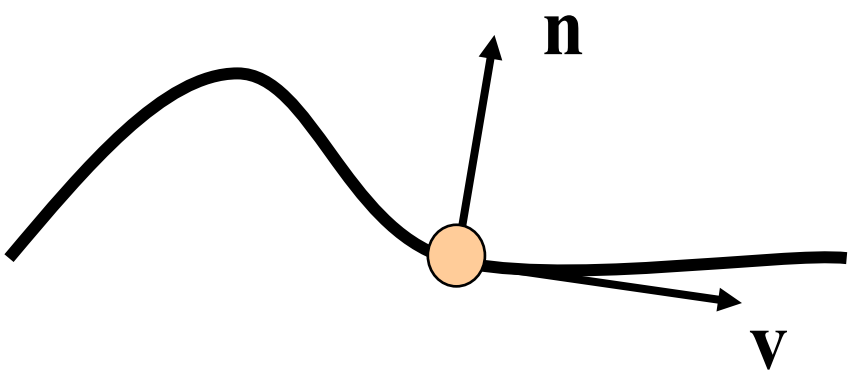
- World is a bunch of **particles**
- Particles interact via forces
  - *Constant*: gravity
  - *Position/time dependent*: force fields
  - *Velocity dependent*: drag
  - *N-ary dependent*: springs, collisions
- Force is function $F(p_1,\ldots,p_n,v_1,\ldots,v_n,t)$
  - Handle with this in a physics engine
  - Insert particles & forces and "turn the crank"





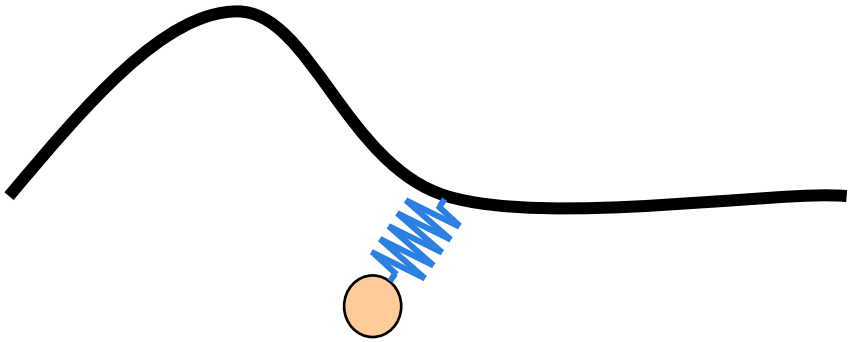
Constrained Particle Behavior

- Suppose we have a bead on a wire
  - The bead can slide freely along wire
  - It can never come off, however hard we pull.
  - How does the bead move under applied forces?
- Usually a curve given by function $\boldsymbol{C}(x,y) = 0$



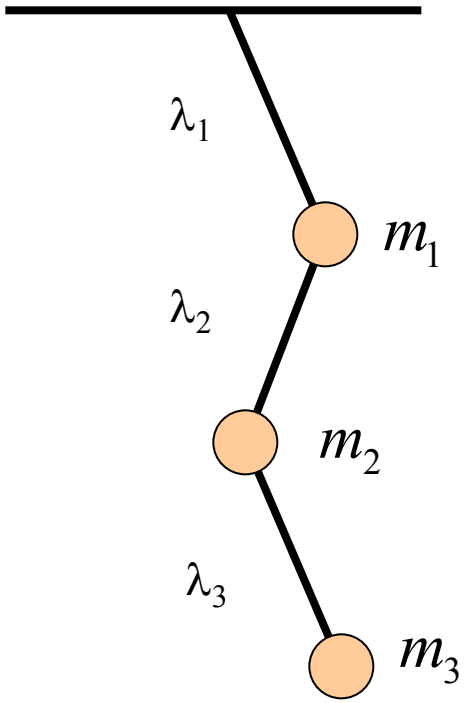
Particle Systems?

- **Idea**: Attach bead to wire with a spring
  - Move the bead normally (maybe off wire)
  - Apply spring force to pull it toward curve
- **Problem**:
  - Weak springs have laggy movement
  - Strong springs have too much energy



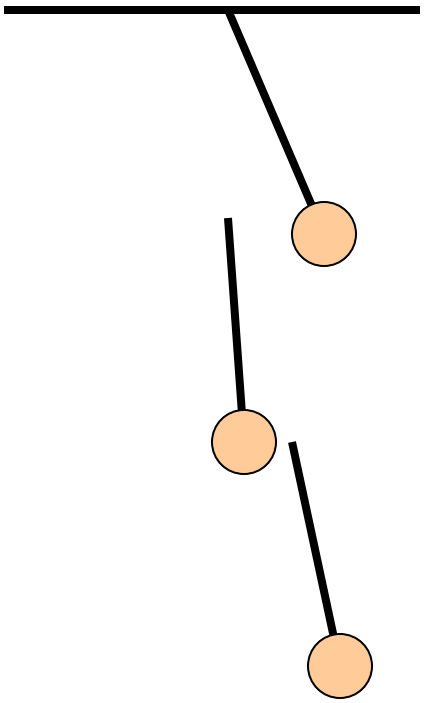
Constraint Solvers

- **Limit** object movement
  - Joints: distance constraint
  - Contact: non-penetration
  - Restitution: bouncing
  - Friction: sliding, sticking
- Many applications
  - Ropes, chains
  - Box stacking
- Focus of Lab 4 (Box2D)



Implementing Constraints

- Very difficult to implement
  - **Errors**: joints to fall apart
  - Called *position drift*
  - Too hard for this course
- Use a physics engine!
  - Box2D supports constraints
  - Limit applications to joints
  - **Example**: ropes, rag dolls
- Want more? CS 5643
  - Or read about it online



Physics in Games

- **Moving** objects about the screen
  - **Kinematics**: Motion ignoring external forces (Only consider position, velocity, acceleration)
  - **Dynamics**: The effect of forces on the screen
- **Collisions** between objects
  - **Collision Detection**: Did a collision occur?
  - **Collision Resolution**: What do we do?

Collisions and Geometry

- Collisions require **geometry**
  - Points are no longer enough
  - Must know *where* objects meet
- Often use convex shapes
  - Lines always remain inside
  - If not convex, call it concave
  - Easiest shapes to compute with
- What to do if is not convex?
  - Break into convex components
  - Triangles are always convex!

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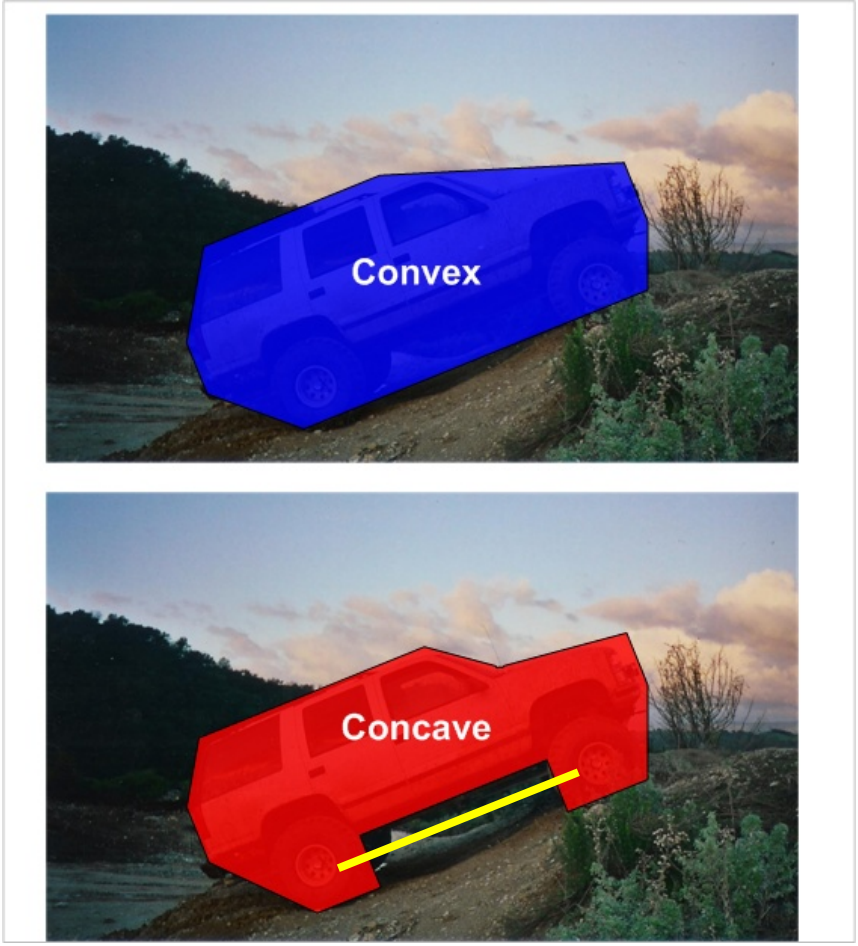
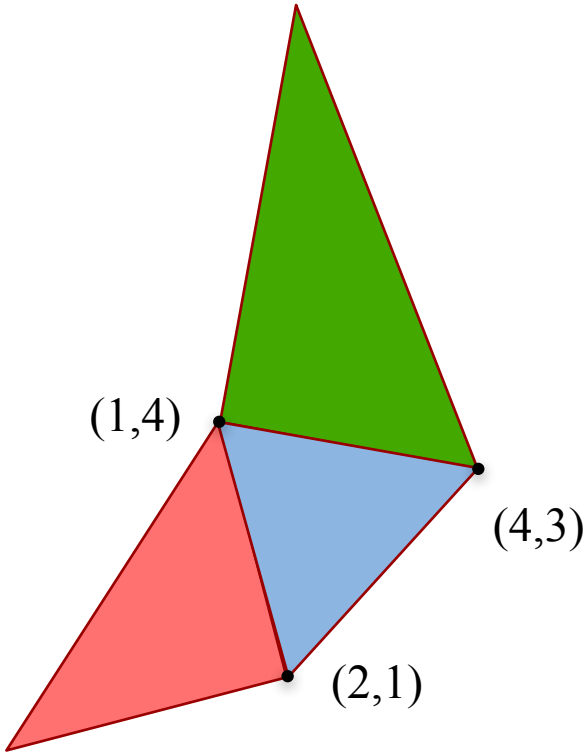


Recall: Triangles in Computer Graphics

- Everything made of **triangles**
  - Mathematically "nice"
  - Hardware support (GPUs)
- Specify with **three vertices**
  - Coordinates of corners
- Composite for complex shapes
  - Array of vertex objects
  - Each 3 vertices = triangle



Recall: Triangles in Computer Graphics

- Everything made of **triangles**
  - Guaranteed to be convex
  - Hardware support (GPUs)
- Specify with **three vertices**
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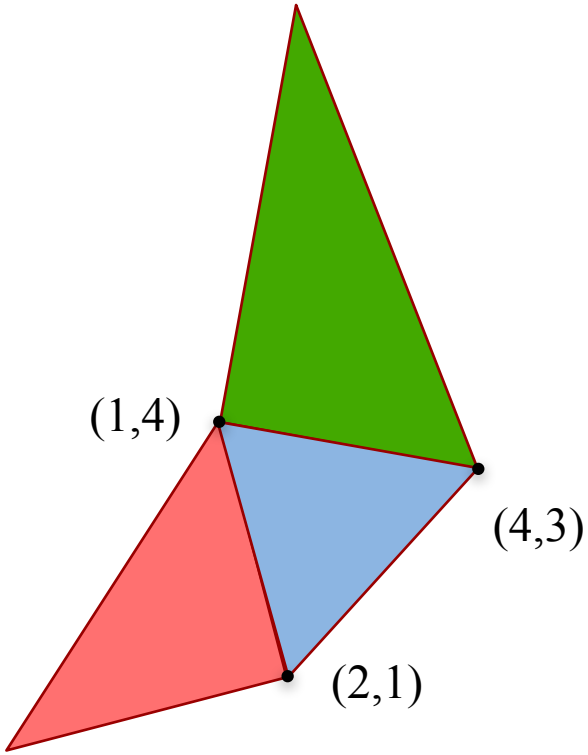

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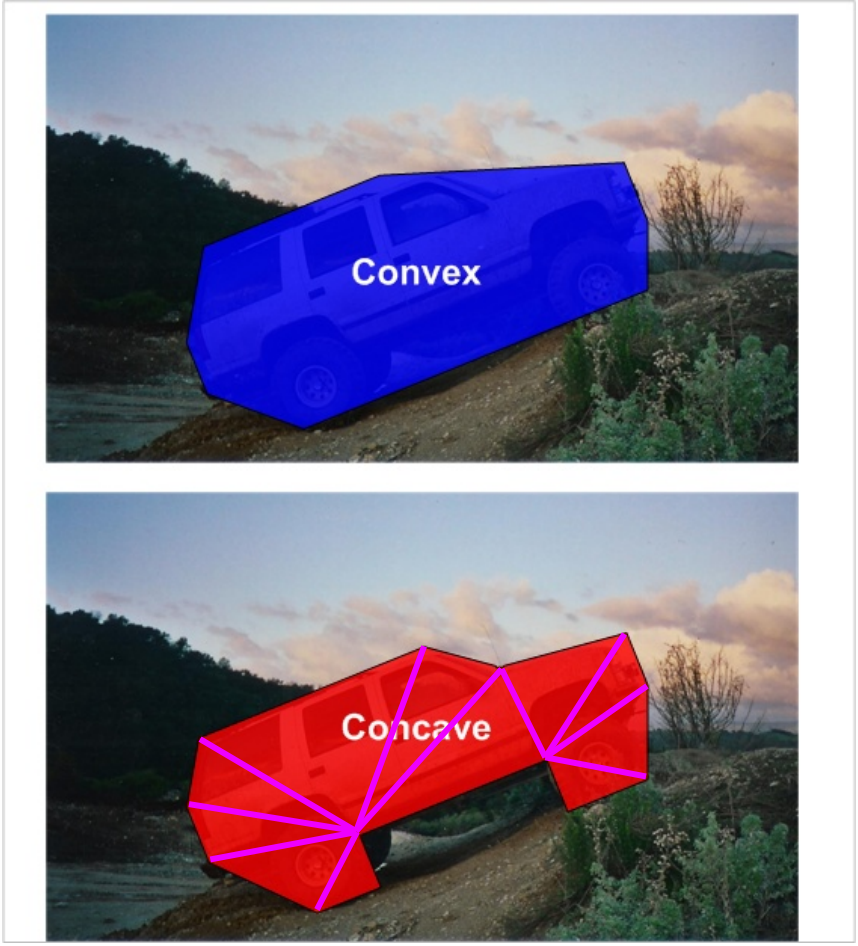
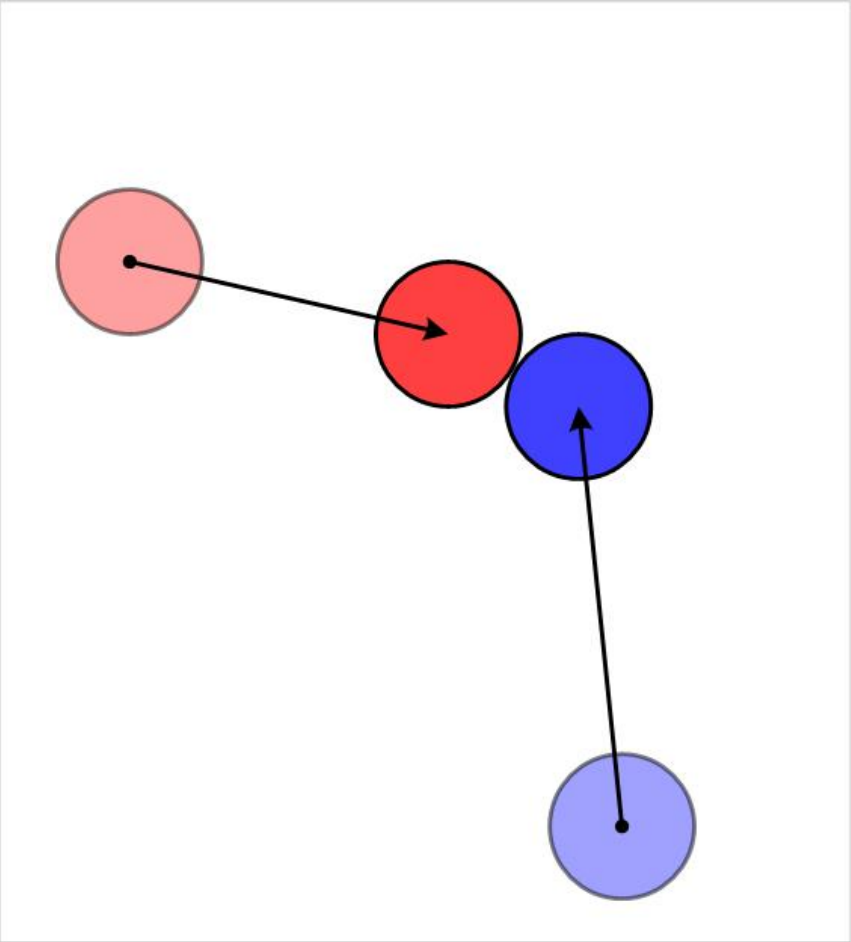


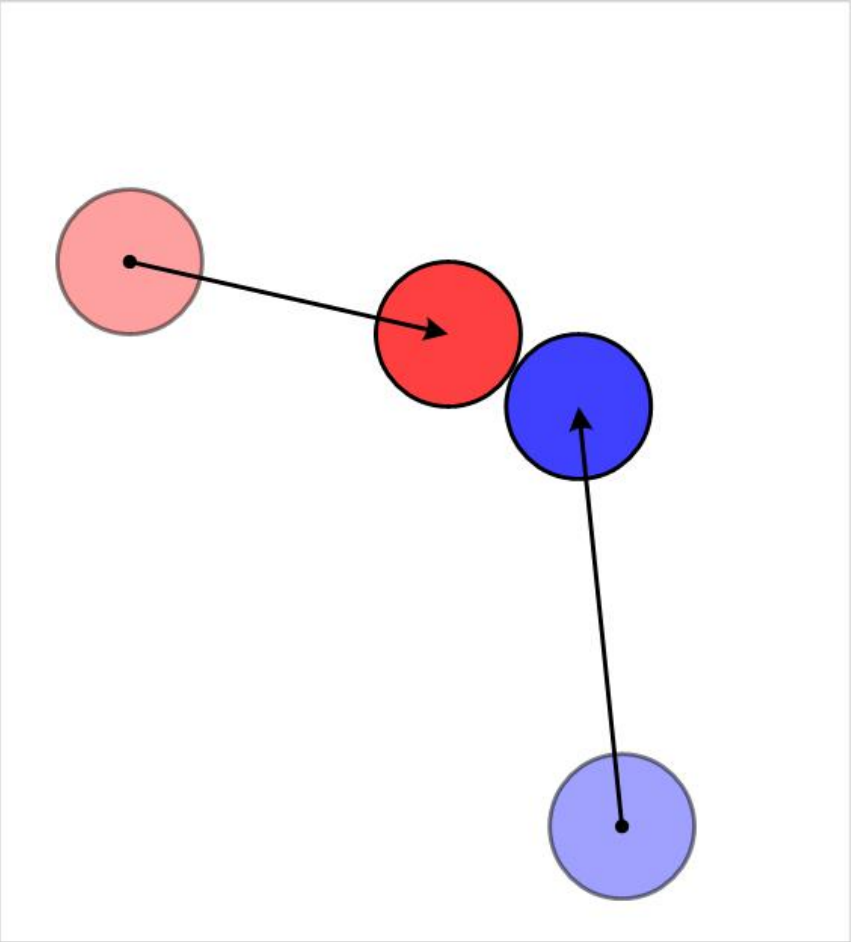
Collision Types

- **Inelastic Collisions**
  - No energy preserved
  - Stop in place ($v = 0$)
  - "Back-out" so no overlap
  - Very easy to implement
- **Elastic Collisions**
  - 100% energy preserved
  - Think billiard balls
  - Classic physics problem



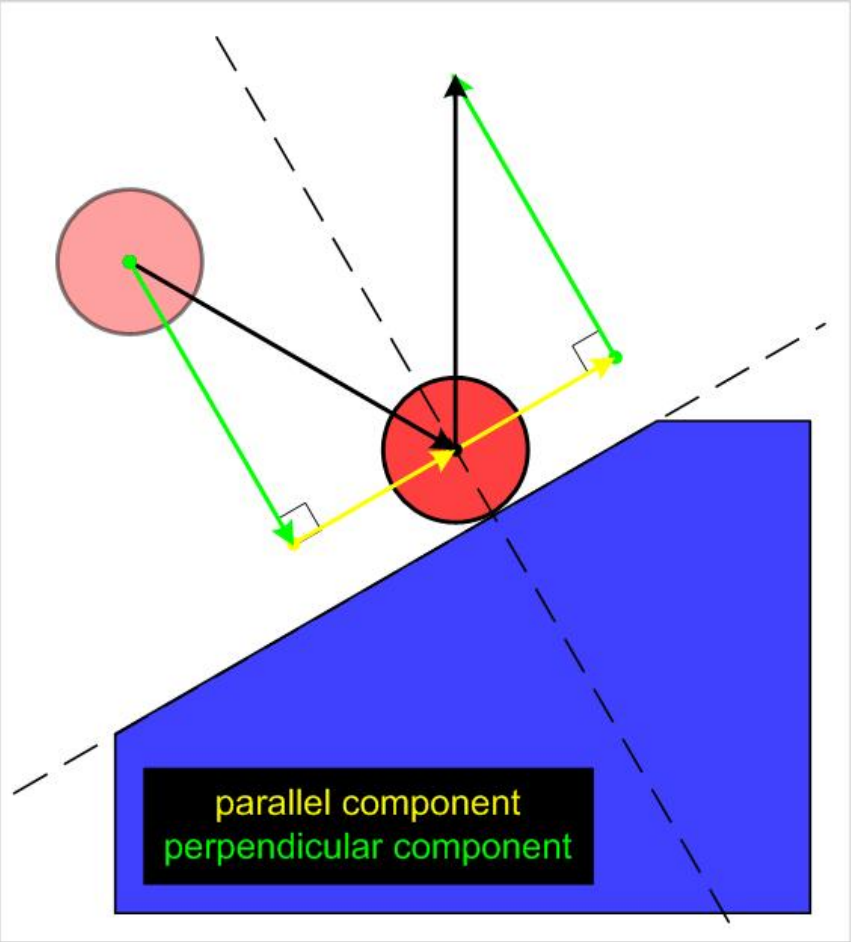
Something In-Between?

- **Partially Elastic**
  - x% energy preserved
  - Different each object
  - Like elastic, but harder
- **Issue**: object "material"
  - What is object made of?
  - **Example**: Rubber? Steel?
- Another parameter!
  - Technical prototype?



Collision Resolution: Circles

- Single point of contact!
  - Energy transferred at point
  - Not true in complex shapes
- Use **relative coordinates**
  - Point of contact is origin
  - **Perpendicular component**: Line through origin, center
  - **Parallel component**: Axis of collision "surface"
- Reverse object motion on the perpendicular comp



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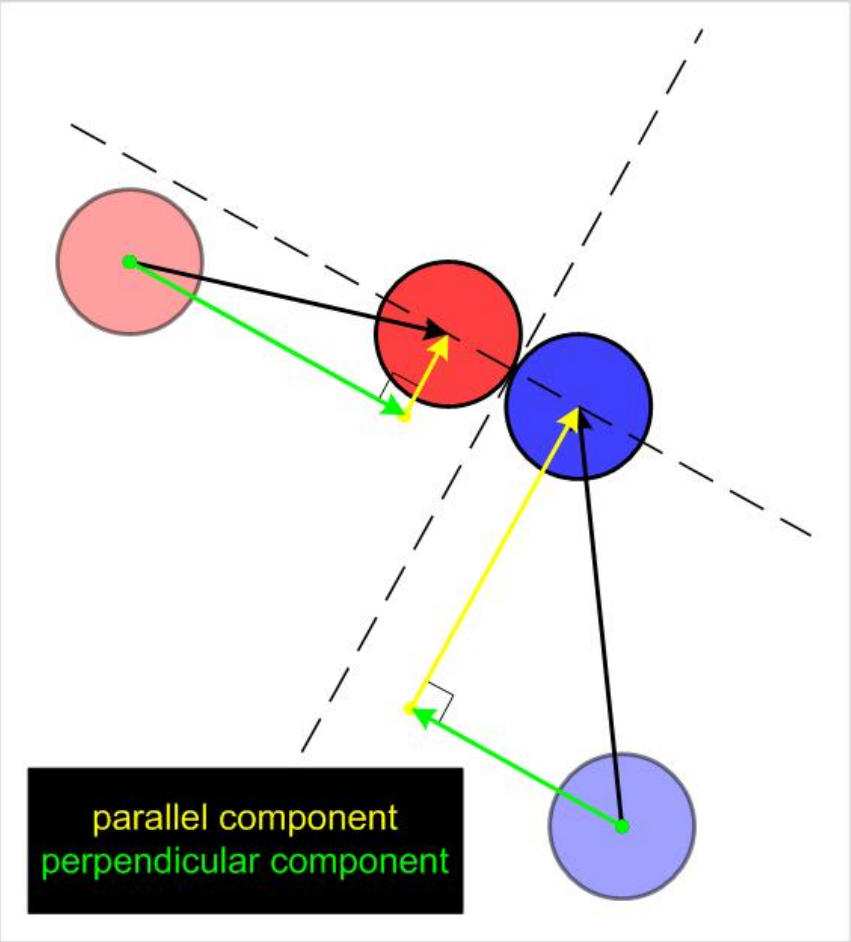
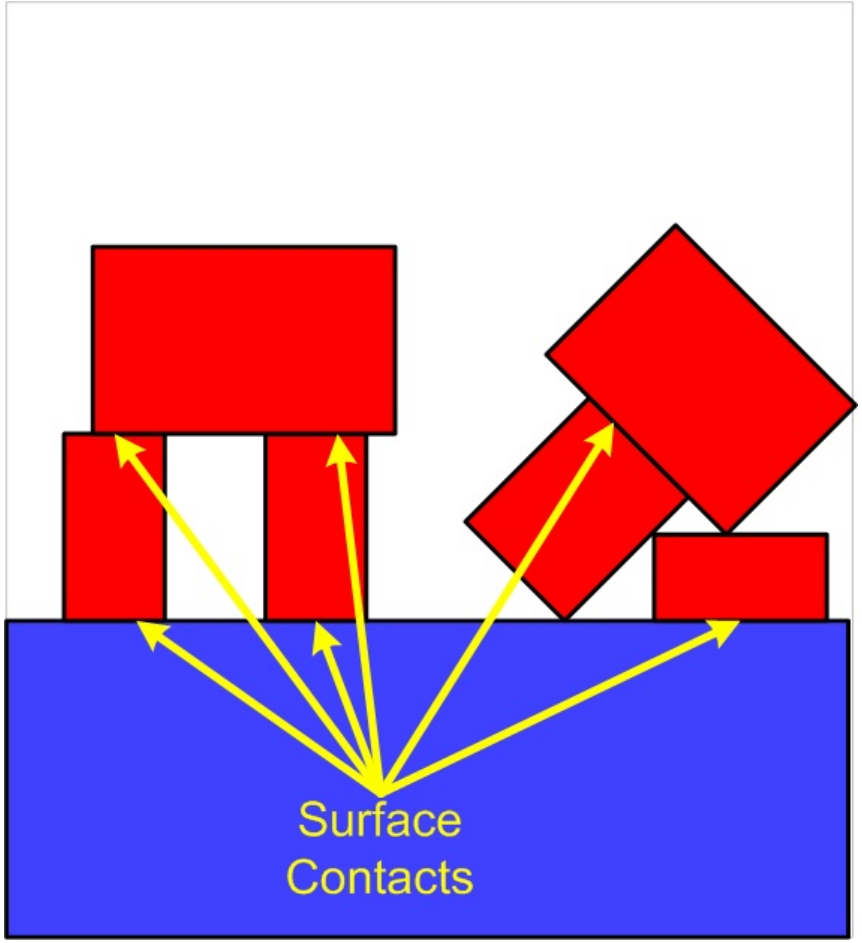
- Use **relative coordinates**
  - Point of contact is origin
  - **Perpendicular component**: Line through origin, center
  - **Parallel component**: Axis of collision "surface"
- **Exchange energy** on the perpendicular comp



More Complex Shapes

- Point of contact harder
  - Could just be a point
  - Or it could be an edge
- Model with **rigid bodies**
  - Break object into points
  - Connect with constraints
  - Force at point of contact
  - Transfers to other points
- Needs **constraint solver**



Summary

- Object representation depends on goals
  - For **motion**, represent object as a **single point**
  - For **collision**, objects must have **geometry**
- Dynamics is the use of forces to move objects
  - **Particle systems**: objects exert a force on one another
  - **Constraint solvers**: restrictions for more rigid behavior
- Collisions are broken up into two steps
  - **Collision detection** checks for intersections
  - **Collision resolution** depends on energy transfer