## State Machines

Problem:
Implement a 3-bit sequence recognizer that produces a high on its output bit when the bit sequence 101 is recognized. Assume that the input is a bit sequence entering from the left one bit at a time:


Mealy Machine: outputs depend on both state and inputs (asynchronous output)


|  | S1 | S0 |
| :---: | :---: | :---: |
| A | 0 | 0 |
| B | 0 | 1 |
| C | 1 | 0 |

On each arc, the label $\mathrm{x} / \mathrm{y}$ means the input is x and the output is y .

| Current State |  | Input | Next State |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S1 | S0 | I | S1 | S0 | Out |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | X | X | X |
| 1 | 1 | 1 | X | X | X |

Using Espresso, the following reduced equations can be obtained:

$$
\begin{aligned}
& S 1^{\prime}=S 0 \cdot I \\
& S 0^{\prime}=I \\
& \text { Out }=S 1 \cdot I
\end{aligned}
$$

Moore Machine: outputs depend only on states


|  | S1 | S0 |
| :---: | :---: | :---: |
| A | 0 | 0 |
| B | 0 | 1 |
| C | 1 | 0 |
| D | 1 | 1 |

Note that in a Moore machine, the outputs are attached to the states, rather than the arcs.

| Current State |  | Input | Next State |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S1 | S0 | I | S1 | S0 | Out |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 |

Using Espresso, the following reduced equations can be obtained:

$$
\begin{aligned}
& \mathrm{S} 1^{\prime}=\mathrm{S} 1 \cdot \underline{\mathrm{~S} 0} \cdot \mathrm{I}+\mathrm{S} 0 \cdot \underline{\mathrm{I}} \\
& \mathrm{~S} 0^{\prime}=\mathrm{I} \\
& \mathrm{Out}^{\prime}=\mathrm{S} 1 \cdot \mathrm{~S} 0
\end{aligned}
$$

