

Combinational Logic

From lecture, we have seen the design of a 1-bit full adder. Now, let's try to add a carry-in bit to the input:

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

The equations can be derived immediately as:

$$\text{Sum} = \underline{a} \underline{b} c + \underline{a} b \underline{c} + a \underline{b} \underline{c} + a b c$$

$$\text{Cout} = \underline{a} b c + a \underline{b} c + a b \underline{c} + a b c$$

To simplify the equations, Karnot Maps should be used:

Sum:

	<u>b</u> <u>c</u>	<u>b</u> c	b <u>c</u>	b c
<u>a</u>	0	1	0	1
a	1	0	1	0

Cout:

	<u>b</u> <u>c</u>	<u>b</u> c	b <u>c</u>	b c
<u>a</u>	0	0	1	0
a	0	1	1	1

While Sum cannot be reduced further, Cout can be reduced to:

$$\text{Cout} = a c + a b + b c$$

## Espresso

Espresso is a logic minimization tool; it is installed on the BSD cluster. To demonstrate it, use the truth table from above. Place the table in a text file as follows (Let's call it adder.raw):

```
.i 3
.o 2
000 00
001 10
010 10
011 01
100 10
101 01
110 01
111 11
.e
```

Don't-cares are indicated as '-' (rather than an 'x' from ECE230).

You can run espresso on this file by executing:

```
espresso -s adder.raw > adder.red
```

The reduced truth table will be saved in adder.red:

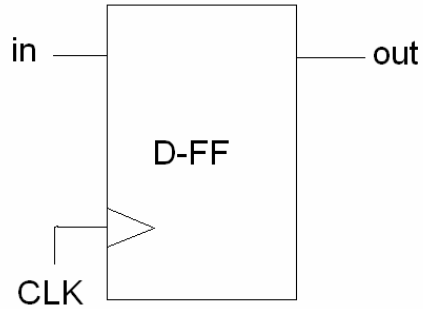
```
.i 3
.o 2
.p 7
100 10
010 10
001 10
111 10
-11 01
1-1 01
11- 01
.e
```

To extract the equations, look at each column on the right (output) side. A '1' indicates that the min-term on the left is included in the reduced equation. For example, the Sum column (1<sup>st</sup> column on the right side) has '1's at the last row (11-), which means that the min-terms is a·b.

(Note: If an equation has more than one min-term, the result is just the "sum" of them)

## Sequential Logic

State holding element: D Flip-flop



On a positive edge of the clock signal, the D Flip-flop passes the input value to the output.

Now, let's implement a 2-bit counter:

Reset	S1	S0	S1'	S0'
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

The equations of S0' and S1' can be derived as:

$$S1' = \underline{\text{Reset}} \underline{S1} \underline{S0} + \underline{\text{Reset}} \underline{S1} \underline{S0}$$

$$S0' = \underline{\text{Reset}} \underline{S1} \underline{S0} + \underline{\text{Reset}} \underline{S1} \underline{S0}$$

And here's a gate diagram of the circuit:

