



CS 311O

Functions

Nate Foster

Spring 2019

Far Above Cayuga's Waters (Dixieland Ramblers)

Review

Previously in 3110:

- **Syntax and semantics**
- **Expressions:** if, let
- **Definitions:** let

Today:

- **Functions**

ANONYMOUS FUNCTION EXPRESSIONS & FUNCTION APPLICATION EXPRESSIONS

Anonymous function expression

Syntax: **fun** **x1** ... **xn** \rightarrow **e**

fun is a keyword :)



Evaluation:

- A function is a value: no further computation to do
- In particular, body **e** is not evaluated until function is applied



Lambda

- Anonymous functions a.k.a. *lambda expressions*
- Math notation: $\lambda x . e$
- The lambda means “what follows is an anonymous function”



Lambda

- [Python](#)
- [Java 8](#)
- A popular [PL blog](#)
- [Lambda style](#)

Functions are values

Can use them **anywhere** we use values:

- Functions can **take** functions as arguments
- Functions can **return** functions as results

This is an incredibly powerful language feature!

Function application

Syntax: $e_0 \ e_1 \ \dots \ e_n$

No parentheses required!

(unless you need to force particular order of evaluation)

Function application

Evaluation of $e_0 \ e_1 \ \dots \ e_n$:

1. Evaluate $e_0 \dots e_n$ to values $v_0 \dots v_n$
2. Type checking will ensure that v_0 is a function **fun** $x_1 \dots x_n \rightarrow e$
3. Substitute v_i for x_i in e yielding new expression e'
4. Evaluate e' to a value v , which is result

Example

Let vs. function

These two expressions are **syntactically different** but **semantically equivalent**:

```
let x = 2 in x+1
```

```
(fun x -> x+1) 2
```

FUNCTION DEFINITIONS

Two ways to define functions

These definitions are **syntactically different** but **semantically equivalent**:

```
let inc = fun x -> x+1
```

```
let inc x = x + 1
```

Fundamentally no difference from **let** definitions we saw before

Recursive function definition

Must explicitly state that function is recursive:

```
let rec f ...
```

Reverse application

- Instead of **f e** can write **e |> f**
- Use: **pipeline** a value through several functions

5 |> inc |> square (* ==> 36*)

assuming

```
let inc x = x + 1
```

```
let square x = x * x
```

FUNCTIONS AND TYPES

Function types

Type $t \rightarrow u$ is the type of a function that takes input of type t and returns output of type u

Type $t1 \rightarrow t2 \rightarrow u$ is the type of a function that takes input of type $t1$ and another input of type $t2$ and returns output of type u

etc.

Note dual purpose for \rightarrow syntax:

- Function types
- Function values

Function application

Type checking:

If $e_0 : t_1 \rightarrow \dots \rightarrow t_n \rightarrow u$

And $e_1 : t_1,$

$\dots,$

$e_n : t_n$

Then $e_0 e_1 \dots e_n : u$

Anonymous function expression

Type checking:

If $x_1:t_1, \dots, x_n:t_n$

And $e:u$

Then $(\text{fun } x_1 \dots x_n \rightarrow e) :$
 $t_1 \rightarrow \dots \rightarrow t_n \rightarrow u$

PARTIAL APPLICATION

More syntactic sugar

Multi-argument functions do not exist

fun x y -> e

is syntactic sugar for

fun x -> (fun y -> e)

More syntactic sugar

Multi-argument functions do not exist

fun x y z -> e

is syntactic sugar for

fun x -> (fun y -> (fun z -> e))

More syntactic sugar

Multi-argument functions do not exist

```
let add x y = x + y
```

is syntactic sugar for

```
let add = fun x ->  
            fun y ->  
              x + y
```

Again: **Functions are values**

Can use them **anywhere** we use values:

- Functions can **take** functions as arguments
- Functions can **return** functions as results

This is an incredibly powerful language feature!

Upcoming events

- [Tomorrow] A0 released by end of day

*This is **fun!***

THIS IS 3110