

# CS 3110

## Functions

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Spring 2018

# A0: Warmup



- Worth only 1% of final grade; other assignments will be 5%
  - much easier coding problems
  - intended to give you low-stakes experience with 3110 workflow
- Please review the **late policy** in the course syllabus
  - sliding scale of penalty based on days late
  - *deadline* is the time by which you must successfully upload your solution files to CMS and confirm that CMS has recorded the correct versions of those files
- Please review the **academic integrity** policy in the course syllabus
  - we use MOSS to detect copying of code; *it works*
  - cite your sources (people, URLs)
  - don't claim other people's ideas/code as your own – that is a violation of AI and will lead to prosecution
- Please don't try to submit by email, regardless of reason

# Waitlist

- Opens now
- Closes tomorrow at midnight!
- <https://goo.gl/forms/HFTOoWRq7uhFehpz2>
- PINs issued (hopefully) Monday
- If you already contacted me in any way about this, you still need to fill out form

# Recitation swap

- Opens today at about noon
- Closes tomorrow at midnight!
- <https://goo.gl/forms/WSutOJW4g2tdcKb73>
- PINs issued (hopefully) Monday
- If you already contacted me in any way about this, you still need to fill out form
- If you filled it out last weekend, your info is still there: please update as needed
- New recitation: TuTh @ 12:20-1:10pm

# Review

Previously in 3110:

- What is a functional language?
- Why learn to program in a functional language?

Today:

- **Functions:** the most important part of functional programming!

# Question

Did you read the syllabus?

A. Yes

B. No

C. I plead the 5<sup>th</sup>

A close-up, front-facing shot of Morpheus from the movie The Matrix. He is bald, has a serious expression, and is wearing his signature black sunglasses. The background is a blurred, dimly lit interior. The text 'WHAT IF I TOLD YOU' is overlaid in large, white, bold, sans-serif font with a black outline at the top of the image.

**WHAT IF I TOLD YOU**

**THE ANSWER IS IN THE SYLLABUS**

# Five aspects of learning a PL

1. **Syntax:** How do you write language constructs?
  2. **Semantics:** What do programs mean? (Type checking, evaluation rules)
  3. **Idioms:** What are typical patterns for using language features to express your computation?
  4. **Libraries:** What facilities does the language (or a third-party project) provide as “standard”? (E.g., file access, data structures)
  5. **Tools:** What do language implementations provide to make your job easier? (E.g., top-level, debugger, GUI editor, ...)
- All are essential for good programmers to understand
  - Breaking a new PL down into these pieces makes it easier to learn



# Our Focus

We focus on **semantics** and **idioms** for OCaml

- **Semantics** is like a meta-tool: it will help you learn languages
- **Idioms** will make you a better programmer in those languages

**Libraries** and **tools** are a secondary focus: throughout your career you'll learn new ones on the job every year

**Syntax** is almost always boring

- A fact to learn, like “**Cornell was founded in 1865**”
- People obsess over subjective preferences {yawn}
- Class rule: **We don't complain about syntax**



**HATERS GONNA HATE**

# Expressions

*Expressions* (aka *terms*):

- primary building block of OCaml programs
- akin to *statements* or *commands* in imperative languages
- can get arbitrarily large since any expression can contain subexpressions, etc.

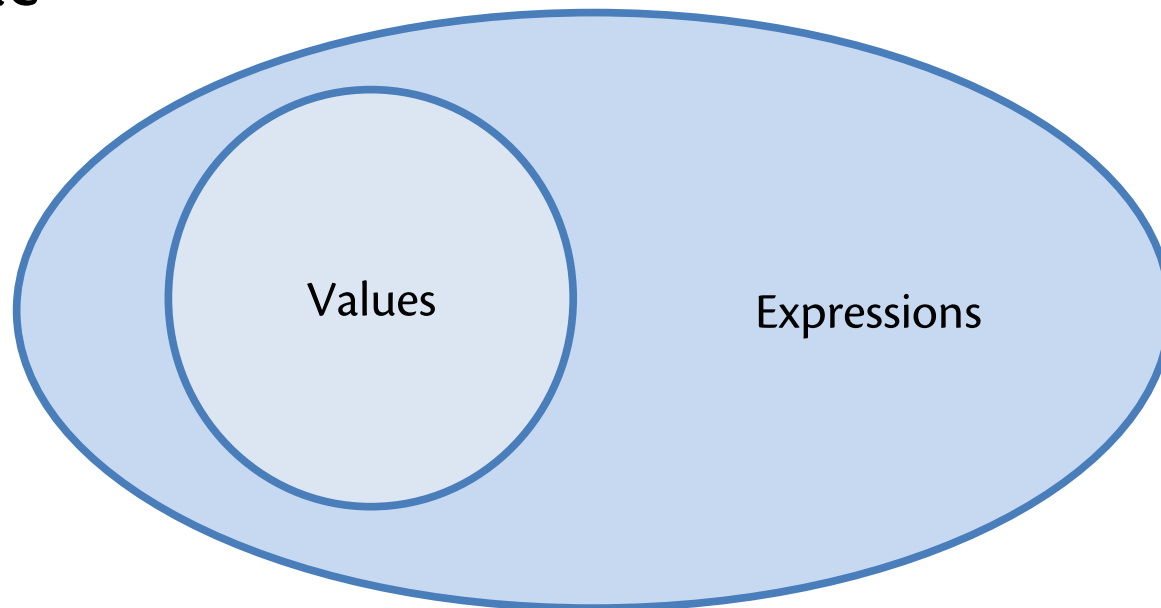
Every kind of expression has:

- **Syntax**
- **Semantics:**
  - **Type-checking rules (*static semantics*):** produce a type or fail with an error message
  - **Evaluation rules (*dynamic semantics*):** produce a *value*
    - (or exception or infinite loop)
    - Used only on expressions that type-check

# Values

A **value** is an expression that does not need any further evaluation

- **34** is a value of type **int**
- **34+17** is an expression of type **int** but is not a value



# **IF EXPRESSIONS**

# if expressions

Syntax:

```
if e1 then e2 else e3
```

Evaluation:

- if **e1** evaluates to **true**, and if **e2** evaluates to **v**, then **if e1 then e2 else e3** evaluates to **v**
- if **e1** evaluates to **false**, and if **e3** evaluates to **v**, then **if e1 then e2 else e3** evaluates to **v**

Type checking:

```
if e1 has type bool and e2 has type t and e3 has type t  
then if e1 then e2 else e3 has type t
```

# Types

Write **colon** to indicate type of expression

As does the top-level:

```
# let x = 22;;
```

```
val x : int = 22
```

Pronounce colon as "has type"

# if expressions

Syntax:

```
if e1 then e2 else e3
```

Evaluation:

- if **e1** evaluates to **true**, and if **e2** evaluates to **v**, then **if e1 then e2 else e3** evaluates to **v**
- if **e1** evaluates to **false**, and if **e3** evaluates to **v**, then **if e1 then e2 else e3** evaluates to **v**

Type checking:

```
if e1 : bool and e2 : t and e3 : t  
then if e1 then e2 else e3 : t
```

# if expressions

Syntax:

```
if e1 then e2 else e3
```

Evaluation:

- if **e1** evaluates to **true**, and if **e2** evaluates to **v**, then **if e1 then e2 else e3** evaluates to **v**
- if **e1** evaluates to **false**, and if **e3** evaluates to **v**, then **if e1 then e2 else e3** evaluates to **v**

Type checking:

```
if e1 : bool and e2 : t and e3 : t  
then (if e1 then e2 else e3) : t
```



# Question

To what value does this expression evaluate?

```
if 22=0 then 1 else 2
```

- A. 0
- B. 1
- C. 2
- D. none of the above
- E. I don't know

# Question

To what value does this expression evaluate?

```
if 22=0 then 1 else 2
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- A. 0
- B. 1
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- D. none of the above
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# Question

To what value does this expression evaluate?

```
if 22=0 then "bear" else 2
```

- A. 0
- B. 1
- C. 2
- D. none of the above
- E. I don't know

# Question

To what value does this expression evaluate?

```
if 22=0 then "bear" else 2
```

- A. 0
- B. 1
- C. 2
- D. none of the above: doesn't type check so never gets a chance to be evaluated; note how this is (overly) conservative
- E. I don't know

# **FUNCTIONS**

# Function definition

Functions:

- Like Java methods, have arguments and result
- Unlike Java, no classes, **this**, **return**

Example *function definition*:

```
(* requires: y>=0 *)  
(* returns: x to the power of y *)  
let rec pow x y =  
  if y=0 then 1  
  else x * pow x (y-1)
```

Note: **rec** is required because the body includes a recursive function call

Note: no types written down! compiler does *type inference*

# Writing argument types

Though types can be inferred, you can write them too.

Parens are then mandatory.

```
let rec pow (x : int) (y : int) : int =  
  if y=0 then 1  
  else x * pow x (y-1)
```

```
let rec pow x y =  
  if y=0 then 1  
  else x * pow x (y-1)
```

```
let cube x = pow x 3
```

```
let cube (x : int) : int = pow x 3
```

# Function definition

Syntax:

```
let rec f x1 x2 ... xn = e
```

note: **rec** can be omitted if function is not recursive

Evaluation:

Not an expression! Just defining the function;  
will be evaluated later, when applied



# Function types

Type  $\mathbf{t} \rightarrow \mathbf{u}$  is the type of a function that takes input of type  $\mathbf{t}$  and returns output of type  $\mathbf{u}$

Type  $\mathbf{t1} \rightarrow \mathbf{t2} \rightarrow \mathbf{u}$  is the type of a function that takes input of type  $\mathbf{t1}$  and another input of type  $\mathbf{t2}$  and returns output of type  $\mathbf{u}$

etc.

# Function definition

Syntax:

```
let rec f x1 x2 ... xn = e
```

Type-checking:

Conclude that  $f : t_1 \rightarrow \dots \rightarrow t_n \rightarrow u$   
if  $e : u$  under these assumptions:

- $x_1 : t_1, \dots, x_n : t_n$  (arguments with their types)
- $f : t_1 \rightarrow \dots \rightarrow t_n \rightarrow u$  (for recursion)

# Function application v1

Syntax: **f e1 ... en**

- Parentheses not required around argument(s)
- Possible for syntax to look like C function call:
  - **f(e1)**
  - if there is exactly one argument
  - and if you do use parentheses
  - and if you leave out the white space

# Function application v1

## Type-checking

if  $f : t_1 \rightarrow \dots \rightarrow t_n \rightarrow u$   
and  $e_1 : t_1, \dots, e_n : t_n$   
then  $f e_1 \dots e_n : u$

e.g.

**pow 2 3 : int**

because **pow : int  $\rightarrow$  int  $\rightarrow$  int**

and **2:int** and **3:int**

# Function application v1

Evaluation of  $f\ e_1 \dots e_n$ :

1. Evaluate arguments  $e_1 \dots e_n$  to values  $v_1 \dots v_n$
2. Find the definition of  $f$   
 $\text{let } f\ x_1 \dots x_n = e$
3. Substitute  $v_i$  for  $x_i$  in  $e$  yielding new expression  $e'$
4. Evaluate  $e'$  to a value  $v$ , which is result

# Example

```
let area_rect w h = w *. h
let foo = area_rect (1.0 *. 2.0) 11.0
```

To evaluate function application:

1. Evaluate arguments  $(1.0 *. 2.0)$  and  $11.0$  to values  $2.0$  and  $11.0$
2. Find the definition of `area_rect`  
`let area_rect w h = w *. h`
3. Substitute in  $w *. h$  yielding new expression  $2.0 *. 11.0$
4. Evaluate  $2.0 *. 11.0$  to a value  $22.0$ , which is result

# Anonymous functions



Something that is *anonymous* has no name

- **42** is an anonymous **int**
- and we can bind it to a name:  
**let x = 42**
- **fun x -> x+1** is an **anonymous function**
- and we can bind it to a name:  
**let inc = fun x -> x+1**

note: dual purpose for `->` syntax: function types, function values

note: **fun** is a keyword :)

# Anonymous functions

Syntax: **fun** **x1** ... **xn** **->** **e**

Evaluation:

- Is an expression, so can be evaluated
- A function *is a value*: no further computation to do
- In particular, body **e** is not evaluated until function is applied

Type checking:

**(fun x1 ... xn -> e) : t1->...->tn->t**  
if **e : t** under assumptions **x1 : t1, ..., xn : tn**



# Anonymous functions

These definitions are **syntactically different** but **semantically equivalent**:

```
let inc = fun x -> x+1
```

```
let inc x = x+1
```

For now, regard as two ways of saying the same thing  
Later, we'll see great uses for anonymous functions!

# Lambda



- Anonymous functions a.k.a. *lambda expressions*
- Math notation:  $\lambda x . e$
- The lambda means “what follows is an anonymous function”
  - $x$  is its argument
  - $e$  is its body
  - Just like `fun x -> e`, but different "syntax"
- You’ll see “lambda” show up in many places in PL, e.g.:
  - PHP: <http://www.php.net/manual/en/function.create-function.php>
  - Python: <https://docs.python.org/3.5/tutorial/controlflow.html#lambda-expressions>
  - Java 8: <https://docs.oracle.com/javase/tutorial/java/javaOO/lambdaexpressions.html>
  - A popular PL blog: <http://lambda-the-ultimate.org/>
  - Lambda style: <https://www.youtube.com/watch?v=Ci48kqp11F8>

# Function application operator

- Infix operator for reverse function application
- Instead of  $\mathbf{f\ e}$  can write  $\mathbf{e\ |>\ f}$
- Run a value through several functions  
 $\mathbf{5\ |>\ inc\ |>\ square\ (*\ 36\ *)}$
- "pipeline" operator

# Functions are values

- Can use them **anywhere** we use values
- Functions can **take** functions as arguments
- Functions can **return** functions as results
  - ...so functions are *higher-order*
- This is not a new language feature; just a consequence of "a function is a value"
- But it is a feature with massive consequences!

# Upcoming events

- [This week] attendance starts for real; you must attend your registered registration section to get credit

*This is **fun!***

**FOR RECITATION**

# Function application v2

Syntax: **e0 e1 ... en**

- **Function to be applied can be an expression**
  - Maybe just a defined function's name
  - Or maybe an anonymous function
  - Or maybe something even more complicated
- **Example:**
  - **(fun x -> x + 1) 2**

# Function application v2

## Type-checking (not much of a change)

if  $e_0 : t_1 \rightarrow \dots \rightarrow t_n \rightarrow u$

and  $e_1 : t_1, \dots, e_n : t_n$

then  $e_0 e_1 \dots e_n : u$



# Function application v2

Evaluation of  $e_0 \ e_1 \ \dots \ e_n$ :

1. Evaluate arguments  $e_1 \ \dots \ e_n$  to values  $v_1 \ \dots \ v_n$

Also evaluate  $e_0$  to a function

**fun  $x_1 \ \dots \ x_n \rightarrow e$**

2. Substitute  $v_i$  for  $x_i$  in  $e$  yielding new expression  $e'$
3. Evaluate  $e'$  to a value  $v$ , which is result

# Function application v2

Evaluation of  $e_0 e_1 \dots e_n$ :

Evaluate  $e_0$  to a function

**fun**  $x_1 \dots x_n \rightarrow e$

Examples:

- $e_0$  could be an anonymous function expression  
**fun**  $x \rightarrow x+1$   
in which case evaluation is immediately done
- $e_0$  could be the name of a defined function  
**inc**  
in which case look up the definition  
**let** **inc**  $x = x + 1$   
and we now know that's equivalent to  
**let** **inc** = **fun**  $x \rightarrow x+1$   
so evaluates to  
**fun**  $x \rightarrow x+1$