



CS 3110

Streams and Laziness

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Fall 2018

Today's music: "Lazy Days" by Enya

Attendance question

What is the type of **f**?

```
let rec f x = f x
```

A. Doesn't compile

B. 'a -> 'a

C. 'a -> 'b

D. unit -> unit

Review

Previously in 3110:

- Functional programming
- Modular programming

Third unit of course: Data structures

Today:

- Streams
- Laziness

INFINITE LISTS

Discussion

How can an infinite length list fit in a finite computer memory?



"Infinite" data structures

- Sequences of numbers: the naturals, primes, Fibonacci, ...
- Data processed by a program: from a file, from the user, from the network
- **Game tree** (for some games):
 - nodes = game positions
 - edges = legal moves

Question

What does `nats` evaluate to?

```
(* [from n] is the infinite list [[n; n+1; ...]] *)  
let rec from n = n :: from (n+1)
```

```
let nats = from 0
```

- A. [0; 1; 2; ...]
- B. Never terminates (infinite loop)
- C. Exception
- D. Stack overflow

aka **infinite lists**, sequences, delayed lists, lazy lists

STREAMS

List representation

```
(** An ['a mylist] is a finite  
list of values of type  
['a]. *)
```

```
type 'a mylist =  
| Nil  
| Cons of 'a * 'a mylist
```

Stream representation?

```
(** An ['a stream] is an infinite  
list of values of type  
['a]. *)
```

```
type 'a stream =  
| Nil  
| Cons of 'a * 'a stream
```

Stream representation?

```
(** An ['a stream] is an infinite  
list of values of type  
['a]. *)
```

```
type 'a stream =  
  | Nil  
  | Cons of 'a * 'a stream
```

Stream representation?

```
type 'a stream =  
  | Cons of 'a * 'a stream
```

Try coding these if possible:

- the stream of 1's
- the stream of natural numbers

Key idea of this entire lecture:

Delay evaluation

Thunk

fun () -> (* a delayed computation *)

Stream representation

```
(** An ['a stream] is an infinite list  
of values of type ['a].
```

```
AF: [Cons (x, f)] is the stream  
whose head is [x] and tail is  
[f()].
```

```
RI: none *)
```

```
type 'a stream =
```

```
Cons of 'a * (unit -> 'a stream)
```

Notation

Write

`<a; b; c; ...>`

to mean stream whose first elements are a, b, c.

Discussion

```
(** [sum <a1; a2; ...> <b1; b2; ...>]  
    is [<a1 + b1; a2 + b2; ...>] *)
```

```
let rec sum  
  (Cons (h_a, tf_a))  
  (Cons (h_b, tf_b))  
=  
?
```

Discussion

(** [map f <a; b; c; ...>] is
[<f a; f b; f c; ...>] *)

let rec map f (Cons (h, tf)) =
?

A CUTE FIBONACCI TRICK

Fibonacci

fibs 1 1 2 3 5 8 ...

Fibonacci

fibs	1	1	2	3	5	8	...
fibs	1	1	2	3	5	8	...

Fibonacci

fibs	1	1	2	3	5	8	...
tl fibs	1	2	3	5	8	13	...

Fibonacci

fibs	1	1	2	3	5	8	...
tl fibs	1	2	3	5	8	13	...
	2	3	5	8	13	21	...

fibs is
1 then
1 then
(fibs + tl fibs)

Fibonacci

```
let rec fibs =  
  Cons(1, fun () ->  
    Cons(1, fun () ->  
      sum fibs (tl fibs)))
```

But try: `take 100 fibs`

Exponential amount of recomputation: regenerate entire prefix of `fibs`, twice, for each element produced

Solution: the Lazy module, covered in textbook

Upcoming events

- N/A

This is judiciously lazy.

THIS IS 3110