CS 3110

Monads

Prof. Clarkson

Fall 2017

Today's music: *Vámanos Pal Monte* by Eddie Palmieri

Review

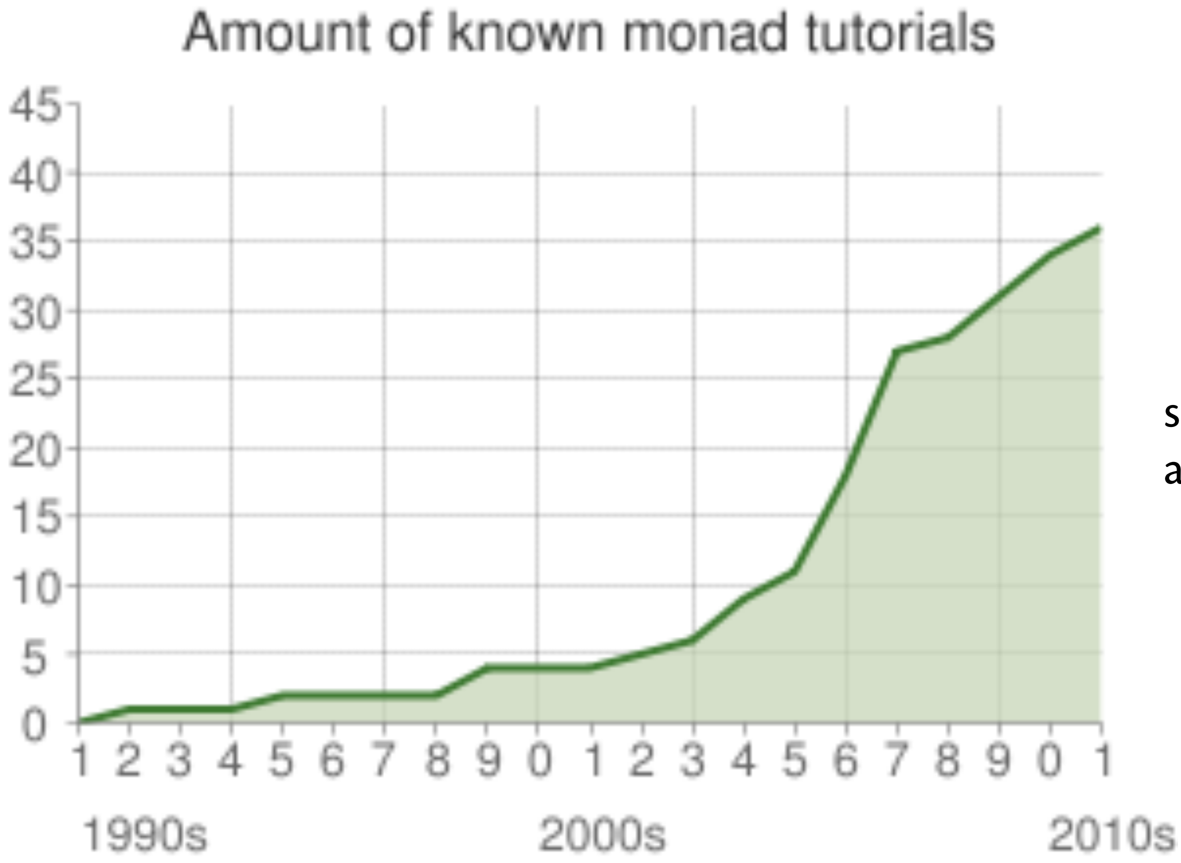
Currently in 3110: Advanced topics

- Futures: Async: deferreds, **`return`**, **`bind`**

Today:

- Monads

Monad tutorials



since 2011:
another 34 at least

source: https://wiki.haskell.org/Monad_tutorials_timeline

Question

Have you programmed with monads in Haskell?

A. No
B. Yes
C. Yes, and I've written a monad tutorial

Monad tutorials

"A monad is a monoid object in a category of endofunctors....It might be helpful to see a monad as a lax functor from a terminal bicategory."

Monad tutor

"A monad is a mon
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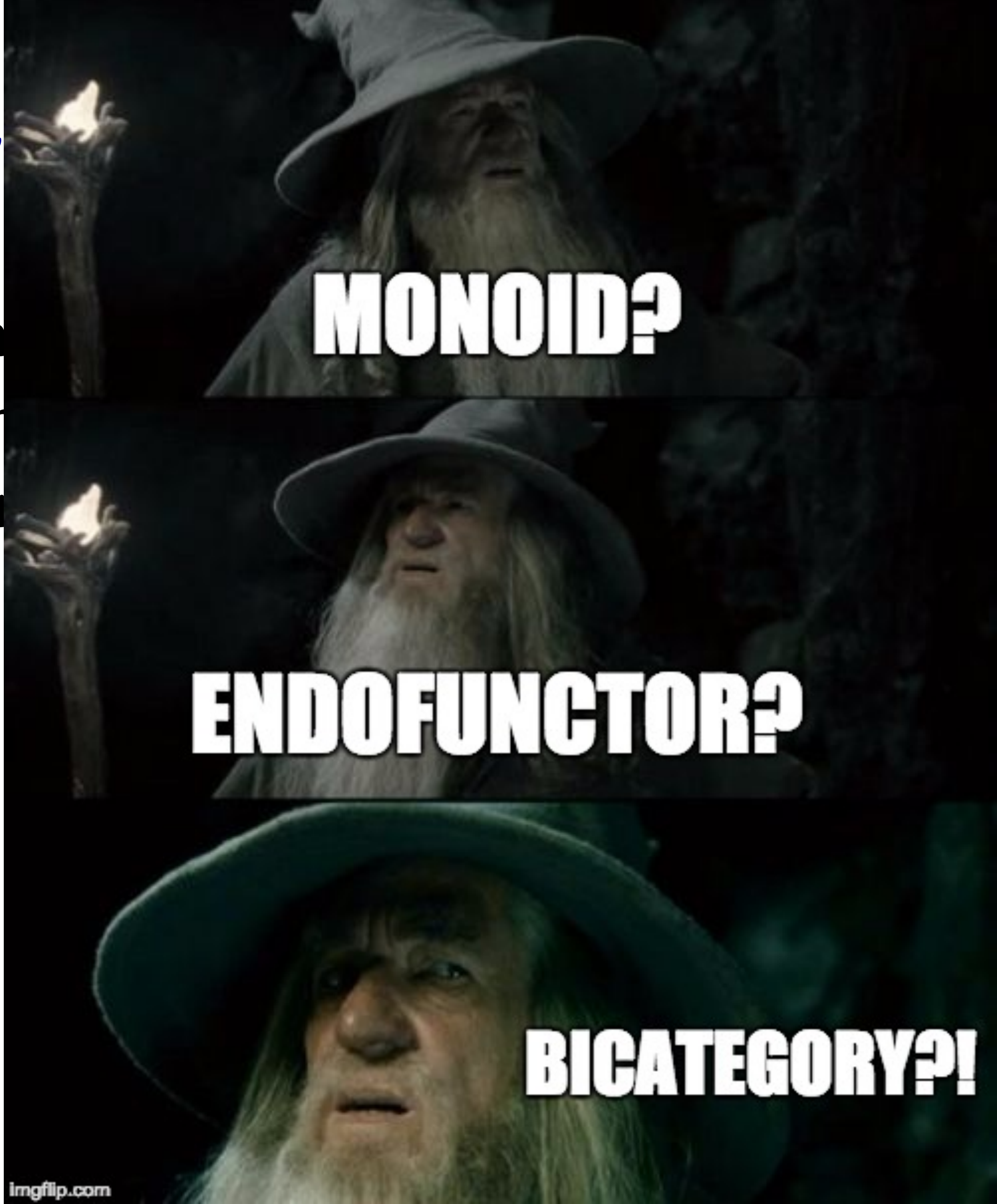


Monad tutorials

"A monad is a monoid object in a category of endofunctors....It might be helpful to see a monad as a lax functor from a terminal bicategory."

"Monads are burritos." [http://chrisdone.com/posts/monads-are-burritos]

Monad

For our purposes:

```
module type Monad = sig
  type 'a t
  val bind   : 'a t -> ('a -> 'b t) -> 'b t
  val return : 'a -> 'a t
end
```

Any structure that implements the **Monad** signature is a monad.

What's the big deal???

LOGGABLE FUNCTIONS

Loggable functions

Suppose you're implementing two functions:

- `f: int -> int`
- `g: int -> int`

And you'd like to compute their *composition*:

```
let h x = g (f x) (* = x |> f |> g *)
let (>>) f g x = x |> f |> g
let h x = (f >> g) x
let h = f >> g
```

Loggable functions

You'd like also log some additional information each time function is called:

- `f_log: int -> int * string`
- `g_log: int -> int * string`

Loggable functions

```
let inc x = x+1
let dec x = x-1
let id = inc >> dec

let inc_log x = (x+1,
  "incremented " ^ string_of_int x ^ "; ")
let dec_log x = (x-1,
  "decremented " ^ string_of_int x ^ "; ")

(* let id_log = inc_log >> dec_log *)
```

Q: Why doesn't that work?

A: `dec_log` takes an **`int`** as input not an **`int * string`**

Loggable functions

```
let id_log x =
  let (y,s1) = inc_log x in
  let (z,s2) = dec_log y in
  (z,s1^s2)
```

Critique:

- Hard to infer from that code that it's doing composition!
- Ugly and verbose compared to
  `let id_log = inc_log >> dec_log`

Upgrading a function

What if we could upgrade a loggable function to accept the input from another loggable function?

```
upgrade f_log
: int*string -> int*string
```

Upgrading a function

```
let upgrade f_log (x,s1) =
  let (y,s2) = f_log x in
  (y,s1^s2)

let id_log =
  inc_log >> upgrade dec_log
```

Nice separation of concerns!

- `upgrade` handles the "plumbing" with the strings
- the definition of `id_log` is clearly about composition

Another kind of upgrade

- Given `f : int -> int`
- How to make it loggable, but with empty log message?
- Need to "lift" a function
  from `int -> int`
  to `int -> int*string`
- That's easy:

```
let trivial x = (x, "")
let lift f = f >> trivial
```

Types

Consider the types:

```
val upgrade :
     (int               -> int * string)
  ->  int * string -> int * string

val trivial :
     int -> (int * string)
```

Types

Another way of writing those types:

```
type 'a t = 'a * string

val upgrade :
      (int    -> int t)
  ->   int t -> int t

val trivial :
      int -> int t
```

Types

Let's swap the argument order of upgrade...

```
val upgrade :
  (int -> int t)
  -> int t
  -> int t

let upgrade' x f = upgrade f x

val upgrade' :
 int t
 -> (int -> int t)
 -> int t
```

Types

```
type 'a t  = 'a * string

val upgrade' :
     int t
  -> (int -> int t)
  -> int t

val trivial :
     int -> int t
```

Have you seen those types before?

Rewriting types

```
type 'a t  = 'a * string

val bind :
      int t
  -> (int -> int t)
  -> int t

val return :
      int -> int t
```

```
module type Monad = sig
  type 'a t
  val bind :
       'a t
    -> ('a -> 'b t)
    -> 'b t
  val return :
    'a -> 'a t
end
```

Loggable is a monad

```
module Loggable : Monad = struct
  type 'a t = 'a * string
  let bind (x,s1) f =
    let (y,s2) = f x in
    (y,s1^s2)
  let return x = (x,"")
end
```

More often called the **writer** monad

Stepping back…

- We took functions
- We made them compute *something more*
  - A logging string
- We invented ways to pipeline them together
  - **`upgrade`**, **`trivial`**
- We discovered those ways correspond to the **`Monad`** signature

FUNCTIONS THAT PRODUCE ERRORS

Functions and errors

- A4: you implemented an interpreter
  - Results could be either values or exceptions
  - So evaluation produced a variant with constructor for either possibility
- A *partial* function (in math) is undefined for some inputs
  - e.g., **`max_list : int list -> int`**
  - with that type, programmer probably intends to raise an exception on the empty list
    - could also produce an option
    - or like A4, could use variant to encode result…

A type for possible errors

```
type 'a t = Val of 'a | Err

let div (x:int) (y:int) =
  if y=0 then Err
  else Val (x / y)

let neg (x:int) = Val (-x)
```

Error handling

Lifting those function to handle inputs that might be errors...

```
let neg_err = function
  | Err -> Err
  | Val x -> Val (-x)

let div_err x y =
  match (x,y) with
  | (Err,_) | (_,Err) -> Err
  | (Val a,Val b) -> if b=0 then Err else Val (a/b)
```

And any other functions you write have to pattern match to handle errors...
Could we get rid of all that boilerplate pattern matching?

Eliminating boilerplate matching

```
(* [rev_app_err m f] applies f
 * to m, like [x |> f], but
 * handling Err as necessary. *)
let rev_app_err m f =
  match m with
    | Val x -> f x
    | Err -> Err

let (|>?) = rev_app_err
```

Eliminating boilerplate matching

```
let neg_err = function
  | Err -> Err
  | Val x -> Val (-x)

let neg_err x =
  x |>? (fun a -> Val (-a))
```

Eliminating boilerplate matching

```
let div_err x y =
  match (x,y) with
  | (Err,_) | (_,Err) -> Err
  | (Val a,Val b) ->
      if b=0 then Err else Val (a/b)

let div_err x y =
   x |>? fun a ->
   y |>? fun b ->
   if b=0 then Err else Val (a/b)
```

Another way to write that code

```
let value x = Val x

let neg_err x =
  x |>? fun a ->
  value (-a)

let div_err x y =
  x |>? fun a ->
  y |>? fun b ->
  if b=0 then Err else value (a/b)
```

What are the types?

```
type 'a t = Val of 'a | Err
val value : 'a -> 'a t
val (|>?) : 'a t -> ('a -> 'b t) -> 'b t
```

Have you seen those types before???

```
module type Monad = sig
  type 'a t
  val bind :
       'a t
    -> ('a -> 'b t)
    -> 'b t
  val return :
     'a -> 'a t
end
```

Error is a monad

```
module Error : Monad = struct
  type 'a t = Val of 'a | Err
  let return x = Val x
  let bind m f =
    match m with
    | Val x -> f x
    | Err -> Err
end
```

Option is a monad

```
module Option : Monad = struct
  type 'a t = Some of 'a | None
  let return x = Some x
  let bind m f =
    match m with
    | Some x -> f x
    | None -> None
end
```

Stepping back...

- We took functions
- We made them compute *something more*
  - A value or possibly an error
- We invented ways to pipeline them together
  - **`value`**, **`(|>?)`**
- We discovered those ways correspond to the **`Monad`** signature

ASYNC

Deferred is a monad

```
module Deferred : sig
  type 'a t
  val return : 'a -> 'a t
  val bind : 'a t -> ('a -> 'b t) -> 'b t
end
```

- `return` takes a value and returns an immediately determined deferred
- `bind` takes a deferred, and a function from a non-deferred to a deferred, and returns a deferred that result from applying the function

Stepping back...

- We took functions
- The Async library made them compute *something more*
  - a deferred result
- The Async library invented ways to pipeline them together
  - **`return`**, **`(>>=)`**
- Those ways correspond to the **`Monad`** signature
- So we call Async a *monadic concurrency library*

Another view of Monad

```
module type Monad = sig
  (* a "boxed" value of type 'a *)
  type 'a t

  (* [m >>= f] unboxes m,
   * passes the result to f,
   * which computes a new result,
   * and returns the boxed new result *)
  val (>>=) : 'a t -> ('a -> 'b t) -> 'b t

  (* box up a value *)
  val return : 'a -> 'a t
end
```

(equate "box" with "tortilla" and you have the burrito metaphor)

SO WHAT IS A MONAD?

Computations

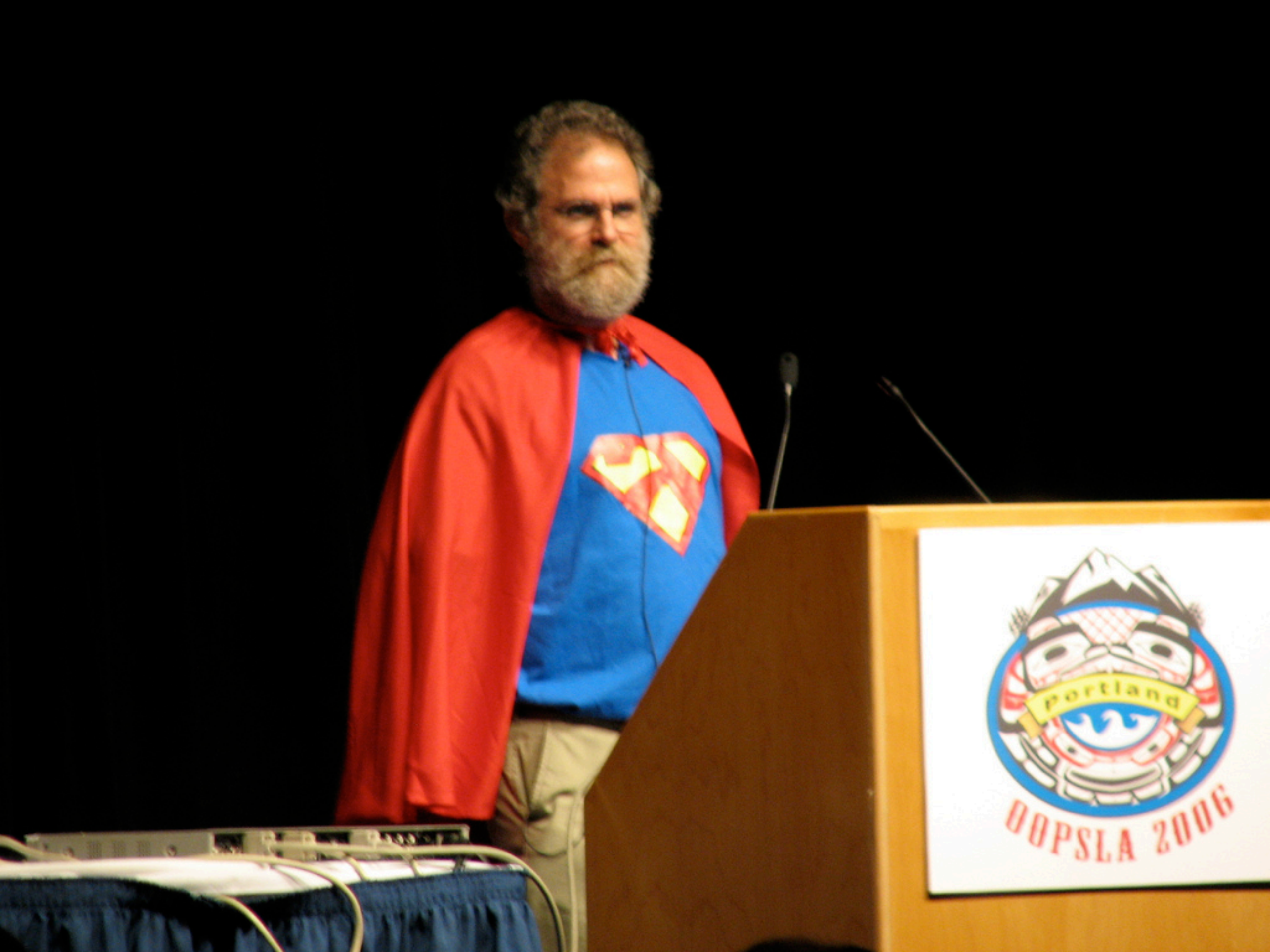
- A *function* maps an input to an output
- A *computation* does that and more: it has some *effect*
  - Loggable computation: effect is a string produced for logging
  - Error computation: effect is a possible error instead of a value
  - Option computation: effect is a possible None instead of a value
  - Deferred computation: effect is delaying production of value until scheduler makes it happen
- A *monad* is a data type for computations
  - **`return`** has the trivial effect
  - **`(>>=)`** does the "plumbing" between effects

Phil Wadler

b. 1956

- A designer of Haskell
- Wrote *the* paper* on using monads for functional programming

* http://homepages.inf.ed.ac.uk/wadler/papers/marktoberdorf/baastad.pdf


portland
OOPSLA 2006

Other monads

- **State:** modifying the state is an effect
- **List:** producing a list of values instead of a single value can be seen as an effect
- **Random:** producing a random value can be seen as an effect
- …

Monad laws

- As you've seen in Coq, data types must obey some algebraic laws
  - e.g., for stacks, **`peek (push x s) = x`**
  - We don't write them in OCaml types, but they're essential for expected behavior
- Monads must obey these laws:
  1. **`return x >>= f`** is equivalent to **`f x`**
  2. **`m >>= return`** is equivalent to **`m`**
  3. **`(m >>= f) >>= g`** is equivalent to **`m >>= (fun x -> f x >>= g)`**
- Why? The laws make sequencing of effects work the way you expect

Monad laws

1. `(return x >>= f) = f x`

   Doing the trivial effect then doing a computation **`f`** is the same as just doing the computation **`f`**

   (return is left identity of bind)

2. `(m >>= return) = m`

   Doing only a trivial effect is the same as not doing any effect

   (return is right identity of bind)

3. ```
   ((m >>= f) >>= g)
    = (m >>= (fun x -> f x >>= g))
   ```

   Doing **`f`** then doing **`g`** as two separate computations is the same as doing a single computation which is **`f`** followed by **`g`**

   (bind is associative)
   ```

Upcoming events

- Happy Thanksgiving Break!
- [Wed and Thur] no class
- [Mon] recitations do meet

This is effectful.

THIS IS 3110