Verification in Coq

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Today's music: *Check Yo Self* by Ice Cube
Review

Previously in 3110:
• Functional programming in Coq
• Logic in Coq
• Curry-Howard correspondence (proofs are programs)
• Induction in Coq

Today: Verification of...
• Functions
• Data structures
• Compilers
Coq for program verification

Coq program

guidance with tactics

Coq theorem

Verified OCaml program

Proof of theorem
Coq for program verification

Coq program

Coq theorem

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Proof of theorem
Coq for program verification

Coq program

Coq theorem

guidance with tactics

Verified OCaml program

Proof of theorem

*This is the hard part*
Theorems and test cases

- Do I have the right ones?
- Do I have enough?
- What am I missing?

... there are no great answers to these questions, only methodologies that help
VERIFICATION OF A FUNCTION

Prove that precondition implies postcondition
Factorial

• **Precondition**: $n \geq 0$

• **Postcondition**: $\text{fact } n = n!$

• **Problem**: how to express $!$ in Coq?
Factorial

Fixpoint fact (n:nat) :=
    match n with
    | 0 => 1
    | S k => n * (fact k)
end.

Theorem fact_correct : forall n, fact n = fact n.
Tail-recursive factorial

Fixpoint fact_tr_acc (n:nat) (acc:nat) :=
  match n with
  | 0 => acc
  | S k => fact_tr_acc k (n * acc)
end.

Definition fact_tr (n:nat) :=
  fact_tr_acc n 1.

Precondition: n >= 0
Postcondition: fact_tr n = fact n
Verify factorial

Lemma helper : forall (n acc : nat),
   fact_tr_acc n acc = (fact n) * acc.
Proof.
   intros n.
   induction n as [ | k IH]; intros acc.
   - simpl. ring.
   - simpl. rewrite IH. ring.
Qed.

Theorem fact_tr_correct : forall n:nat,
   fact_tr n = fact n.
Proof.
   intros n. unfold fact_tr. rewrite helper. ring.
Qed.
Verify factorial

Lemma helper : forall (n acc : nat),
  fact_tr_acc n acc = (fact n) * acc.
Proof.
  intros n.
  induction n as [ | k IH]; intros acc.
  - simpl. ring.
  - simpl. rewrite IH. ring.
Qed.

Theorem fact_tr_correct : forall n:nat,
  fact_tr n = fact n.
Proof.
  intros n. unfold fact_tr. rewrite helper. ring.
Qed.
Extract verified factorial

Extract Inductive nat
=> int [ "0" "succ" ].
Extract Inlined Constant Init.Nat.mul
=> "(*)".
Extraction "fact.ml" fact_tr.

Coq nat becomes OCaml int
Coq * becomes OCaml *
Extract Coq to OCaml
Prove that equations hold for operations in a data structure.
module type Stack = sig

  type 'a t
  val empty : 'a t
  val is_empty : 'a t -> bool
  val size : 'a t -> int
  val peek : 'a t -> 'a option
  val push : 'a -> 'a t -> 'a t
  val pop : 'a t -> 'a t option

end
Categories of operations

• **Creator**: creates value of type "from scratch" without any inputs of that type

• **Producer**: takes value of type as input and returns value of type as output

• **Observer**: takes value of type as input but does not return value of type as output

• **(Mutator)**: takes value of type as input and mutates the value
module type Stack = sig
  type 'a t
  val empty : 'a t
  val is_empty : 'a t -> bool
  val size : 'a t -> int
  val peek : 'a t -> 'a option
  val push : 'a -> 'a t -> 'a t
  val pop : 'a t -> 'a t option
end
Stack eqn. specification

- `is_empty empty = true`
- `is_empty (push _) = false`
- `peek empty = None`
- `peek (push x _) = Some x`
- `size empty = 0`
- `size (push _ s) = 1 + size s`
- `pop empty = None`
- `pop (push _ s) = Some s`
Equational specification

• aka *algebraic specification*
• Set of equations
• Describes interactions between:
  – observers and creators
  – observers and producers
  – producers and creators
  – producers and other producers
• Might not have equation for every possible interaction, because some might not be meaningful
Stack as list

Module MyStack.

Definition stack (A:Type) := list A.

Definition empty {A:Type} : stack A := nil.

Definition is_empty {A:Type} (s : stack A) : bool :=
    match s with
    | nil => true
    | _::_ => false
    end.
Stack as list

Definition push {A:Type} (x : A) (s : stack A) : stack A :=
   x::s.

Definition peek {A:Type} (s : stack A) : option A :=
   match s with
      | nil => None
      | x::_ => Some x
   end.
Stack as list

Definition pop \{A: Type\} (s : stack A)
  : option (stack A)
:=
   match s with
   | nil => None
   | _::xs => Some xs
end.

Definition size \{A: Type\} (s : stack A)
  : nat
:=
   length s.

End MyStack.
Theorem empty_is_empty : forall (A:Type),
    @is_empty A empty = true.
Proof. auto. Qed.

Theorem push_not_empty : forall (A:Type) (x:A) (s : stack A),
    is_empty(push x s) = false.
Proof. auto. Qed.

Theorem peek_empty : forall (A:Type),
    @peek A empty = None.
Proof. auto. Qed.

Theorem peek_push : forall (A:Type) (x:A) (s : stack A),
    peek(push x s) = Some x.
Proof. auto. Qed.
Verify stack as list

Theorem pop_empty : forall (A:Type),
    @pop A empty = None.
Proof. auto. Qed.

Theorem pop_push : forall (A:Type) (x:A) (s : stack A),
    pop(push x s) = Some s.
Proof. auto. Qed.

Theorem size_empty : forall (A:Type),
    @size A empty = 0.
Proof. auto. Qed.

Theorem size_push : forall (A:Type) (x:A) (s : stack A),
    size(push x s) = 1 + size s.
Proof. auto. Qed.
Extract verified stack

Extract Inductive bool => "bool" [ "true" "false" ].
Extract Inductive option => "option" [ "Some" "None" ].
Extract Inductive list => "list" [ "[]" "(::)" ].
Extract Inductive nat => int [ "0" "succ" ].

Extraction "stacks.ml" MyStack.

Coq bool, option, list, nat become OCaml equiv.
Prove that meaning is preserved

VERIFICATION OF A COMPILER
Expressions

Inductive expr : Type :=
| Const : nat -> expr
| Plus : expr -> expr -> expr.

Fixpoint eval_expr (e : expr) : nat :=
match e with
| Const n => n
| Plus e1 e2 =>
  plus (eval_expr e1) (eval_expr e2)
end.
Stack programs

Inductive instr : Type :=
  | PUSH : nat -> instr
  | ADD : instr.

Definition prog := list instr.

Definition stack := list nat.
Stack programs

Fixpoint eval_prog
    (p : prog) (s : stack)
    : option stack
:=

  match p, s with
  | (PUSH n)::p', s =>
    eval_prog p' (n::s)
  | ADD::p', x::y::s' =>
    eval_prog p' ((x+y)::s')
  | nil, s => Some s
  | _, _ => None
end.
Fixpoint compile (e : expr) : prog :=
  match e with
  | Const n => [PUSH n]
  | Plus e1 e2 =>
    compile e2 ++ compile e1 ++ [ADD]
  end.
Verify the compiler

Theorem compile_correct :
  forall (e:expr),
  eval_prog (compile e) []
  = Some [eval_expr e].

Proof in lecture code.
Extract verified compiler

Extract Inlined Constant app => "(@)". Extraction "compiler.ml" compile.
Upcoming events

• [Thur or Fri] A5 out
• [Thur – Tue] Design review meetings

This is verified.

THIS IS 3110