CS3110 Spring 2016 Lecture 23: Distributed Computing with Functional Processes continued

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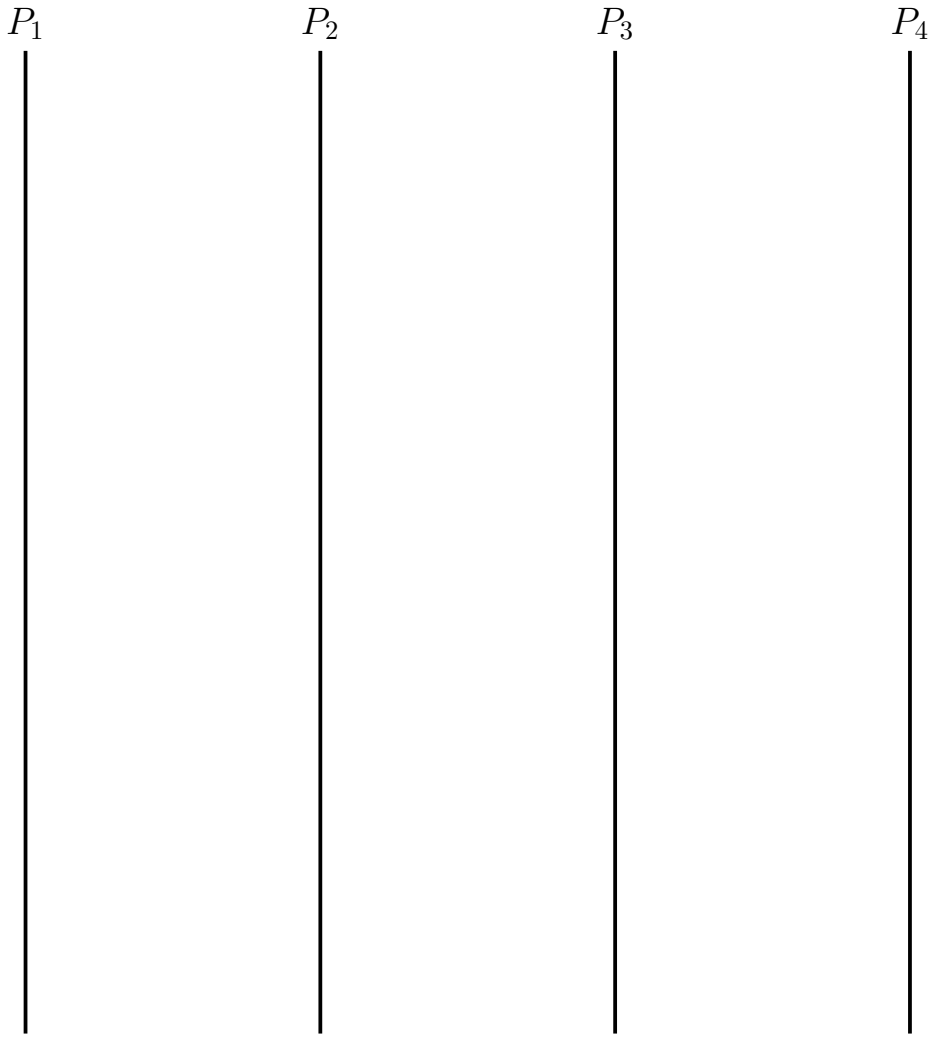
Topics

1. *Message sequence diagrams* and event structures.
2. A General Process Model (GPM) for asynchronous distributed computing.
3. Coding 2/3 consensus as a functional process.
4. A Logic of Events for reasoning about protocols.

Reminders

1. Dr. Yaron Minksy will give the next lecture on Tuesday May 3.
2. PS6 is about distributed computing in OCaml, in particular implementing the 2/3 simple consensus protocol. The staff has created a substantial new software system for your use.

1 Message sequence diagrams for consensus – ($3f + 1$ case)



2 General Process Model (GPM)

2.1 Overview

To reason about protocols we need a precise yet realistic mathematical model, like the evaluation rules for OCaml programs. Adding rules for sending and receiving messages is more complex than the evaluation rules. We need to formalize properties of the *communication infrastructure*. At one extreme is communication over the internet and at the other, communication in a data center.

At a very high level of abstraction, the communication layer introduces a level of *uncertainty*. This includes a certain amount of *randomness* combined with a loss of information due to complexity. All of the precise formal models of communication are limited in various ways. This leads to a plethora of formal models such as Petri nets (one of the oldest), communicating sequential processes (CSP, Hoare [4]), Calculus of Communicating Systems (CCS, Milner [9]), IO Automata (Lynch [8]), Message Automata ([1, 3]), and many more.

Here we use a very simple method called a General Process Model (GPM [2]) tailored to treat asynchronous protocols.

We will not study this model in detail (even though it is very precisely defined). Here are the basic ideas presented informally. The goal is to be more precise about the notion of an *asynchronous system*.

A system consists of *components*. A component has a *location*, an address for sending it messages. It has a *internal* part which is the computation process. It also has an *external part* used for communication. Communication is done with *messages*. These can hold data and processes.

A system computes in an *environment* which provides communication among components. (There can be communication among systems as well, but we focus on analyzing specific systems.)

2.2 Basic Concepts

The following section is taken from [2].

We begin with an overview of our model of distributed computation and the concepts we use to reason about them. The next section formalizes in Computational Type Theory (CTT) all the italicized terms.

A *system* consists of a set of *components*. Each component has a *location*, an *internal* part, and an *external* part. Locations are just abstract identifiers. There may be more than one component with the same location.

The internal part of a component is a *process*—its program and internal (hidden) state. The external part of a component is its interface with the rest of the system. In this paper, this interface will be a list of *messages*, containing either *data* or a process, each labeled with the location of its recipient. The "higher order" ability to send a message containing a process allows a system to grow by "forking" or "bootstrapping" new components. (The external part can also be used to model the shared memory accessible to components at the same location, but will not be discussed in this paper.)

A system computes in steps as follows. In each step, the *environment* may choose and remove a message from the external part of a component. If components exist at the location to which the message is addressed, each of them receives the message as input and computes a pair consisting of a process, which becomes the next internal part of the component, and a list of messages, which is appended to the current external part of the component. If the chosen message is addressed to a location that is not yet in the system, then a *boot process* creates a new component at that location. The boot process to be used is supplied as a system parameter.

An infinite sequence of steps, starting from a given system and using a given boot-process, is a *run* of that system. From a run of a system we derive an abstraction of its behavior by focusing on the *events* in the run. The events are the pairs, $\langle x, n\rangle$, of a location and a step (a "point in space-time") at which location x gets an input message at step n (i.e."information is transferred"). Every event has a location, and there is a natural *causal-ordering* on the set of events, the ordering first considered by Lamport [7]. This allows us to define an *event-ordering*, a structure, $\langle E,\ loc,\ <,\ info\rangle$, in which the causal ordering $<$ is transitive relation on E that is well-founded, and locally-finite (each event has only finitely many predecessors). Also, the events at a given location are totally ordered by $<$. The information, $info(e)$, associated with event e is the message input to $loc(e)$ when the event occurred.

2.3 Leader Election in a Ring

We have found that requirements for distributed systems can be expressed as (higher-order) logical propositions about event-orderings. To illustrate this and motivate the results in the rest of the paper we present a simple example of *leader election* in a group of *processes arranged in a ring*.

Each participating component will be a member of one or more groups and each group has a name. A message $\langle G, j\rangle$ from the environment to component i informs it that it is in group G and has neighbor j in group G. We assume that, by the time the protocol begins, each such group is a ring, that is, the graph of the relation $j = \mathit{neighbor}(G, i)$ is a simple cycle. When any component in a group G receive a message $\langle [\mathit{start}], G\rangle$ it starts the leader election protocol whose goal is to choose one member of group G to be the leader and inform every member of G of the leader's location (presumably as the first step in a more complex protocol). To make this easy we also assume that each component at location i has a unique identifier $\mathit{uid}(i)$ that is a number—so that the uid's can be ordered.

The simple protocol is this: every component that receives a start message proposes itself by sending, to its neighbor, its uid in a message with header *propose*. Every component that receives a proposal with a uid, p, different from its own uid, u, proposes the maximum, $\max(u, p)$ to its neighbor. A component i that receives its own uid in a proposal is the leader and so sends a message with its location, i, and header *leader*. Every component other than the leader that receives a leader message forwards the message to its neighbor.

We describe protocols like this by classifying the events in the protocol. The events in this protocol are the start events, the propose events and the leader events. The components can recognize events in each of these classes (in this example they all have distinctive headers) and they can associate information with each event (e.g. the group G, the proposed uid, the location of the leader). Events in some classes cause events with related information content in other classes.

In general, an *event class* X is function on events in an event ordering that partitions the events into two sets, $E(X)$ and $E - E(X)$, and assigns a value $X(e)$ to events $e \in E(X)$. In our example, let us suppose that the list xs contains the locations of all the components that are participating in the protocol and might be members of the groups. An event e that is the receipt of a start message $\langle [\mathit{start}], G\rangle$ at a location $i \in xs$ is a member of an

event class *Start*, with value $Start(e) = G$. Such classes, defined by a list of locations and a particular message header, are the *basic event classes*. Likewise, we may define basic classes *Propose* and *Leader* with values of the form $Propose(e) = \langle G, p\rangle$ and $Leader(e) = \langle G, x\rangle$. When an event in any of these basic classes occurs, the receiving component, at location $i \in xs$, will be able to associate additional pieces of information with the event, such as its $uid(i)$, or its location i, or $neighbor(G, i)$ from the most recent message from the environment. For example, we define event class $Start^+$ to have the same events as class *Start* but assign values given by

$$\begin{array}{rcl} Start^+(e) & = & \langle G, uid(i), j\rangle \\ \textbf{where} & & i = loc(e),\ j = neighbor(G, i) \end{array}$$

Similarly, we define $Propose^+$, and $Leader^+$ by

$$\begin{array}{rcl} Propose^+(e) & = & \langle G, p, i, uid(i), j\rangle \\ Leader^+(e) & = & \langle G, x, i, j\rangle \end{array}$$

To describe the leader election protocol in terms of these event classes, we declare that every event e with $Start^+(e) = \langle G, uid, j\rangle$ causes an event e' with location j and value $Propose(e') = \langle G, uid\rangle$. Every event e with $Propose^+(e) = \langle G, p, i, uid, j\rangle$ for which $p \neq uid$ causes an event e' with location j and value $Propose(e') = \langle G, \max(p, uid)\rangle$. Every event e with $Propose^+(e) = \langle G, p, i, uid, j\rangle$ for which $p = uid$ causes an event e' with location j and value $Leader(e') = \langle G, i\rangle$. Every event e with $Leader^+(e) = \langle G, x, i, j\rangle$ for which $x \neq i$ causes an event e' with location j and value $Leader(e') = \langle G, x\rangle$.

Clearly, these constraints (and the assumption that group G forms a ring) imply that after a *Start* event, the member $\max \in G$ with the maximum $uid_{\max}$ must eventually propose $uid_{\max}$ and this will be proposed by all members of the group, until component max receives its own $uid_{\max}$. It will then cause a *Leader*-event with value $\langle G, \max\rangle$ at its neighbor and this will be forwarded around the ring, so every member of the group is informed of the location max. The formal proof of these statements is easily constructed using standard logical methods. (If we want to be sure that all *Leader*-events for G have the same value, then we also need constraints that say that *Propose* and *Leader* events are caused *only* by the above rules.)

Programmable classes Each event class in this example is *programmable*. A class X is programmable if at each location l there is a

process that can recognize X-events at l and compute their values using only information received at l.

We describe distributed computation by defining programmable event classes and specifying their interactions in term of *propagation rules* and *propagation constraints*.

Propagation rules and constraints If A and B are event classes, the *propagation rule* $A \stackrel{f}{\Rightarrow} B@g$ is a proposition about event orderings saying that for every A-event with value v, there is a B-event, with value $f(v)$, causally after it, at each location $x \in g(v)$. We require that distinct A-events cause distinct B-events. Formally,

$$\begin{aligned}\forall x\colon \mathit{Loc}.\ \exists p\colon \{e\colon E(A) | x \in g(A(e))\} \\ \rightarrow \{e'\colon E(B) | \mathit{loc}(e') = x\}. \\ \mathit{injection}(p)\ \wedge \\ \forall e\colon E(A).\ e < p(e)\ \wedge\ B(p(e)) = f(A(e))\end{aligned}$$

where injection(p) asserts that that the function p is one-to-one.

The *propagation constraint* $A \stackrel{f}{\Leftarrow} B@g$ is the same proposition, but with $\mathit{injection}(p)$ replaced by $\mathit{surjection}(p)$. This says that every B-event "comes from" an appropriate A-event.

We can express our leader election protocol as a conjunction of propagation rules and constraints. For instance, two of the propagation rules are:

$$\begin{aligned}&\mathit{Start}^+ \stackrel{f}{\Rightarrow} \mathit{Propose}@g,\ \textbf{where} \\ &\quad f(\langle G, \mathit{uid}, j\rangle) = \langle G, \mathit{uid}\rangle, \quad g(\langle G, \mathit{uid}, j\rangle) = [j] \\ &\mathit{Leader}^+ \stackrel{f}{\Rightarrow} \mathit{Leader}@g,\ \textbf{where} \\ &\quad f(\langle G, x, i, j\rangle) = \langle G, x\rangle \\ &\quad g(\langle G, x, i, j\rangle) = \text{if } x = i \text{ then } \mathit{nil} \text{ else } [j]\end{aligned}$$

If ψ is a proposition about event orderings, we say that a system *realizes* ψ if the event-ordering of any run of the system satisfies ψ. We extend the "proofs-as-programs" paradigm to "proofs-as-processes" for distributed computing by making constructive proofs that requirements are *realizable*. For compositional reasoning, it is desirable to create, when possible, a *strong realizer* of requirement ψ—a system that realizes ψ in any context. Formally, system S is a strong realizer of ψ if the event-ordering of any run of a system S' such that $S \subseteq S'$, satisfies ψ. If S_1 is a strong realizer of ψ_1

and S_2 is a strong realizer of ψ_2, then $S_1 \cup S_2$ is a strong realizer of $\psi_1 \wedge \psi_2$.

One of our main results is that we can automatically extract strong realizers for propagation rules like those used in the leader election example. Basic event classes are programmable, and the set of programmable event classes is closed under a variety of *combinators*.

If B is a basic class and if we have *reliable message delivery*, then a component may cause an event in B by placing a message with an appropriate header on its external part. A rule, $A \Rightarrow B$ is *programmable-basic* (PB) if A is programmable and B is basic. Thus, under the assumption of reliable message delivery, every PB-rule is realizable.

Reliable message delivery is an assumption about the environment. One weakening of this assumption allows some components to suffer *send omission faults*. Under this assumption, parameterized by a set of locations, F, called the *fail-set*, every message on the external part of a component whose location is not in F, will eventually be delivered.

If send omissions are allowed, not every PB-rule is realizable, but the restricted rule $A|(\neg F) \Rightarrow B$ is realizable, when $A \Rightarrow B$ is PB, and $A|(\neg F)$ is the class of A-events whose location is not in the fail-set. A fault-tolerant protocol like Paxos can be described by such restricted rules, and proved correct under appropriate assumptions on the size of the fail-set.

A PB-rule $A \Rightarrow B$ is also strongly realizable. This is because, essentially by definition, class A is programmable only if there is a system S that recognizes A-events in any context. So in a run of system, S', with $S \subseteq S'$, the components in S will still recognize A-events. Also, if message delivery is reliable then the addition of extra components will not prevent the required B-events from occurring.

Unfortunately, some desirable properties of protocols like leader election do not follow from conjunctions of PB-rules alone. We also need some propagation constraints, of the form $A \overset{f}{\Leftarrow} B@g$. The realizer we construct for $A \overset{f}{\Rightarrow} B@g$ generates B-events only from A-events, so it also realizes the propagation constraint $A \overset{f}{\Leftarrow} B@g$. But it is not a strong realizer of the constraint because, in an unrestricted larger system, other components may cause B-events.

Strong realizers will always compose to strong realizers. We can compose (nonstrong) realizers for propagation rules and propagation constraints if

we can show that they do not "interfere" with one another. For example, the realizer we construct for $A \Rightarrow B$ will trivially realize $C \Leftarrow D$ if classes B and D are disjoint; and that can be trivially guaranteed if classes B and D are basic classes distinguished by different "message headers." That simple design rule reduces the proof of noninterference to a "compatibility check" that the message headers used by different rules are different.

Because a verified system may run in an environment that includes unverified, untrusted code for which we cannot perform the compatibility check, it would be desirable to ensure that the verified system is a strong realizer for a conjunction of propagation rules and propagation constraints. One way to modify a group of components in a realizer so that they form a strong realizer is to have them encrypt their messages with a shared key.

3 Coding 2/3 Consensus

Various languages have been designed to facilitate coding asynchronous protocols and systems. These systems have a special character because they interact with an environment which is fundamentally unpredictable. We do not have room in this course to deeply explore these languages and formalisms that were mentioned already in this lecture. However, it is worth a brief look at the style of one such language developed here at Cornell called EventML [10]. It is similar in spirit to the Orc language (or orchestration) developed at the University of Texas [5, 6]. Here is a sample of code and definitions in EventML, taken as excerpts from this paper [11]

References

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