CS 3110

Type Inference

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Fall 2016

Today's music: Cool, Calm, and Collected by The Rolling Stones

Review

Previously in 3110:

- Interpreters: ASTs, evaluation, parsing
- Formal syntax
- Formal semantics
  - Small-step
  - Big-step

Today:

- Type inference

Kinds of typing

- **Static**: type checking done by analysis of program
  - Compiler/interpreter verifies that type errors cannot occur
  - e.g., C, C++, F#, Haskell, Java, OCaml
- **Dynamic**: type checking done by run-time
  - Run-time detects type errors and report them. Usually requires keeping extra *tag* information for each value in memory.
  - e.g., JavaScript, LISP, Matlab, PHP, Python, Ruby
- Can be a spectrum, e.g., **`instanceof`** in Java: some checking done at compile time, rest of checking done at run time

Kinds of typing

- **Strong**: type of a value is independent of how it's used
  - Can't pass a `string` where an `int` expected, etc.
  - e.g., OCaml, Haskell, Python, Java, Ruby
- **Weak**: type of value is dependent on how it's used
  - If a `string` is used where an `int` expected, it gets converted automatically or by type cast to an `int`
  - e.g., C, C++, Perl
- Can be a spectrum
  - e.g., Java **+** operator converts objects to strings
- Troll alert: strong vs. weak is debated a lot; probably not helpful to degenerate into such debates

Typing quadrant

	Weak	Strong
Static	C, C++	OCaml, Java, Haskell
Dynamic	Perl, Assembly	Ruby, Python, Scheme

Kinds of typing

- **Manifest**: type information supplied in source code
  - e.g., C, C++, Java
- **Implicit**: type information not supplied in source code
  - Implementation 1: Dynamic typing
    - e.g., LISP, Python, Ruby, PHP
  - Implementation 2: Type inference
    - e.g., Haskell, OCaml
  - Tradeoff: ease of implementation vs. run-time performance
- Can be a spectrum
  - e.g., no reasonable language requires you to write to provide the type of **5** in `x:int = 5`

Type inference

- Goal is to reconstruct types of expressions based on known types of some symbols that occur in expressions
  - Type checkers have to do some of this anyway
  - Difference between inference and checking is really a matter of degree
- Best known in functional languages
  - Especially useful in managing the types of higher-order functions
  - But starting to appear in mainstream languages, e.g., C++11:
    - **`auto x = e;`** declares variable **`x`**, initialized with expression **`e`**, and type of **`x`** is automatically inferred
    - **`decltype(e)`** is a type that means "whatever type **`e`** has"
- Invented by Robin Milner for SML (though other people also deserve credit; see the notes)

Robin Milner

1934-2010

Awarded 1991 Turing Award for
"…ML, the first language to include polymorphic type inference and a type-safe exception handling mechanism…"

Is type inference hard?

- The algorithm used in ML is quite clever yet relatively easy to implement
- Difficulty of doing type inference for any particular language is often hard to determine
- Designing type inference for a particular language can be quite hard; must balance
  - expressivity of type system with
  - possibility of inferring all types without requiring annotations

HM type inference

- Algorithm used in OCaml is called HM
  - Hindley & Milner invented it independently

- Guarantees of HM:
  - **It never makes mistakes.** HM will never infer types that cause a program to fail to type check.
  - **It never fails.** HM will never reject a program that could have been type-checked if programmer had written down all the types.
    - (true of *nearly* all the language; over time some features have been added for which it's not true; see RWO for examples)

HM type inference

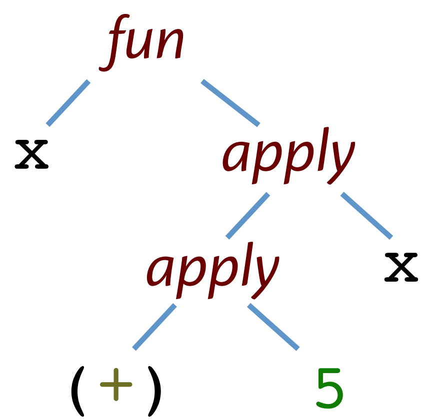
- Determine types of definitions in order
  - Use types of earlier definitions to infer later
  - (which is one reason why you can't use later definitions in file)
- For each definition:
  - collect *constraints* on types
  - solve constraints to determine type

Example

```
let g x = 5 + x
```

Desugar:

```
let g = fun x -> ((+) 5) x
```



Example

```
let g = fun x -> ((+) 5) x
```

Step 1: Assign preliminary types to all subexpressions

Subexpression	Preliminary type
`fun x -> ((+) 5) x`	

Example

```
let g = fun x -> ((+) 5) x
```

Step 1: Assign preliminary types to all subexpressions

Subexpression	Preliminary type
`fun x -> ((+) 5) x`	
`x`	
`((+) 5) x`	
`(+) 5`	
`(+)`	
`5`	
`x`	

Example

```
let g = fun x -> ((+) 5) x
```

Step 1: Assign preliminary types to all subexpressions

Subexpression	Preliminary type
`fun x -> ((+) 5) x`	
`x`	
`((+) 5) x`	
`(+) 5`	
`(+)`	`int -> int -> int`
`5`	`int`
`x`	

Example

```
let g = fun x -> ((+) 5) x
```

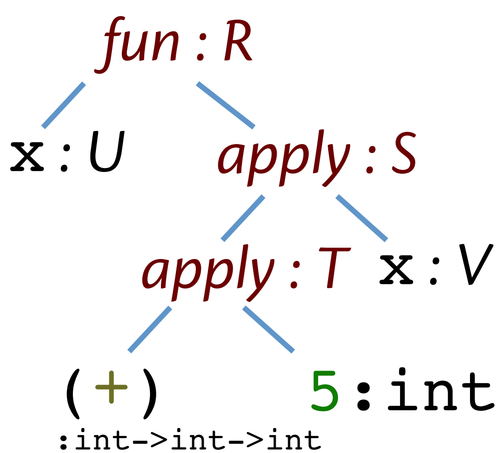
Step 1: Assign preliminary types to all subexpressions

Subexpression	Preliminary type
`fun x -> ((+) 5) x`	*R*
`x`	*U*
`((+) 5) x`	*S*
`(+) 5`	*T*
`(+)`	`int -> int -> int`
`5`	`int`
`x`	*V*

R,S,T,U,V are preliminary type variables used during inference

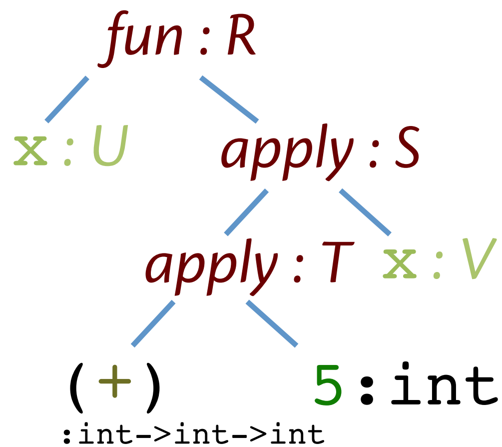
Example

Subexpression	Preliminary type
`fun x -> ((+) 5) x`	*R*
`x`	*U*
`((+) 5) x`	*S*
`(+) 5`	*T*
`(+)`	`int -> int -> int`
`5`	`int`
`x`	*V*



Question

Did we really need to give **`x`** two different preliminary type variables?

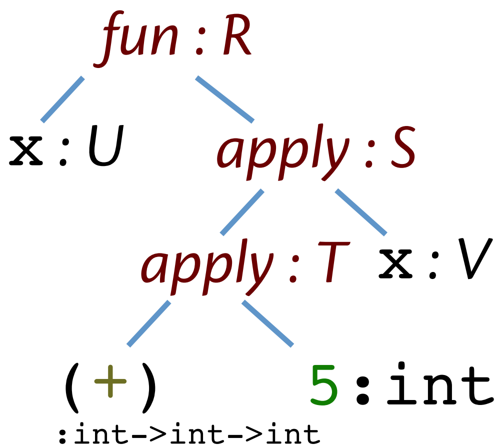


A. Yes

B. No

Question

Did we really need to give **`x`** two different preliminary type variables?



A. Yes

B. No

Example

```
let g = fun x -> ((+) 5) x
```

Step 2: Collect constraints

Example

```
let g = fun x -> ((+) 5) x
```

Step 2: Collect constraints

Subexpression	Preliminary type
`fun x -> ((+) 5) x`	R
`x`	U
`((+) 5) x`	S

Constraint from function:

$$R = U \to S$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 2: Collect constraints

Subexpression	Preliminary type
`x`	U
`x`	V

Constraint from variable usage:

$$U = V$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 2: Collect constraints

Subexpression	Preliminary type
`((+) 5) x`	S
`x`	V
`(+) 5`	T

Constraint from application:

$T = V \texttt{->} S$

Example

```
let g = fun x -> ((+) 5) x
```

Step 2: Collect constraints

Subexpression	Preliminary type
`(+) 5`	T
`(+)`	`int -> int -> int`
`5`	`int`

Constraint from application:

`int -> int -> int` = `int ->` T

Example

```
let g = fun x -> ((+) 5) x
```

Step 2: Collect constraints

$$
\begin{array}{rcl}
U & = & V \\
R & = & U \texttt{->} S \\
T & = & V \texttt{->} S \\
\texttt{int -> int -> int} & = & \texttt{int ->}\ T
\end{array}
$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 3: Solve constraints

$$\begin{array}{rcl}
U & = & V \\
R & = & U \texttt{->} S \\
T & = & V \texttt{->} S \\
\texttt{int -> int -> int} & = & \texttt{int ->}\ T
\end{array}$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 3: Solve constraints

$$
\begin{array}{rcl}
U & = & V \\
R & = & U \texttt{->} S \\
T & = & V \texttt{->} S \\
\texttt{int -> int -> int} & = & \texttt{int ->}\ T
\end{array}
$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 3: Solve constraints

$$
\begin{array}{rcl}
R & = & U \texttt{->} S \\
T & = & U \texttt{->} S \\
\texttt{int -> int -> int} & = & \texttt{int -> } T
\end{array}
$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 3: Solve constraints

$$\begin{array}{rcl} R & = & U \texttt{->} S \\ T & = & U \texttt{->} S \\ \texttt{int -> int -> int} & = & \texttt{int ->}\ T \end{array}$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 3: Solve constraints

$$R = U \texttt{->} S$$

$$\texttt{int -> int -> int} = \texttt{int ->}\ U\ \texttt{->}\ S$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 3: Solve constraints

$$R = U \texttt{->} S$$

$$\texttt{int -> int -> int} = \texttt{int ->} U \texttt{->} S$$

Example

```
let g = fun x -> ((+) 5) x
```

Step 3: Solve constraints

R = **`int -> int`**

Example

```
let g = fun x -> ((+) 5) x
```

Step 3: Solve constraints

R = `int -> int`

Done: type of **`g`** is **`int -> int`**

Algorithm for constraint collection

- **Input:** an expression **e**
  - Assume that every anonymous function in **e** has a different variable name as its argument
  - Easy to ensure that holds, thanks to lexical scope: rename arguments as necessary
- **Output:** a set of constraints

Constraint collection

- Intuition: assign a unique type variable (e.g., *R*, *S*, *T*, …),
  - one to each argument **`x`** of a function in **`e`**
  - one to every subexpression **`e'`** in **`e`**
  - like how we decorated (aka annotated) AST in example

- Formally: define two functions that return type variables:
  - D: *definition* of an argument
  - U: *use* of a subexpression
  - D(**`x`**) returns the type variable assigned to argument **`x`**
  - U(**`e'`**) returns the type variable assigned to subexpression **`e'`**

Constraint collection

Example:

- Input: **`fun x -> (fun y -> x)`**
- Define two functions for type variables:
  - D(**`x`**) = *R*
  - D(**`y`**) = *S*
  - U(**`fun x -> (fun y -> x)`**) = *T*
  - U(**`fun y -> x`**) = X
  - U(**`x`**) = *Y*

Constraint collection

Constraints that are collected (intuition):

- For each kind of expression (application, anonymous function, let, etc.), collect a set of equations that must hold for that kind of expression
  - **e.g.**, the type of entire anonymous function must equal type of its argument *arrow* type of its body
  - which is what we did in example earlier

Constraint collection

Constraints that are collected (formally):

- At a variable usage **`x`**:
  U(**`x`**) = D(**`x`**)
- At a function application **`e1 e2`**:
  U(**`e1`**) = U(**`e2`**) **`->`** U(**`e1 e2`**)
- At an anonymous function **`fun x -> e`**:
- U(**`fun x -> e`**) = D(**`x`**) **`->`** U(**`e`**)
- At a let expression **`let x = e1 in e2`**:
  U(**`let x = e1 in e2`**) = U(**`e2`**) and D(**`x`**) = U(**`e1`**)
- etc.
- Unioned with constraints collected at each subexpression
- Note how these are essentially the static semantics!

Return those constraints as output of algorithm

Constraint collection

Example (continued):

- Input: **`fun x -> (fun y -> x)`**
- **`x`** occurs as subexpression, so generate constraint U(**`x`**) = D(**`x`**)
  - Already have U(**`x`**) = *Y* and D(**`x`**) = *R*
  - So constraint is *Y* = *R*
- **`fun y -> ux`** occurs as subexpression, so generate constraint U(**`fun y -> x`**) = D(**`y`**) **->** U(**`x`**)
  - Already have U(**`x`**) = *Y*, and U(**`fun y -> x`**) = *X*, and D(**`y`**) = *S*
  - So constraint is *X* = *S* **->** *Y*
- **`fun x -> (fun y -> x)`** occurs as subexpression, so generate constraint U(**`fun x -> (fun y -> x)`**) = D(**`x`**) -> U(**`fun y -> x`**)
  - Resulting constraint is *T* = *R* **->** *X*

Solving constraints

- After collection, have a set of constraints
  - Really a set of equations
- Need to solve those equations for type of main expression of interest
- *Unification* algorithm [Robinson 1965]
  - roughly like Gaussian elimination to solve system of matrix equations in linear algebra
  - see notes for the algorithm

Upcoming events

- nothing this week

This is cool, calm, and collected.

THIS IS 3110