

$$\begin{array}{c}
\frac{x_1 : P(a) \wedge \neg P(a)}{P(a)} \text{ (\wedge EL)} \qquad \frac{\frac{x_1 : P(a) \wedge \neg P(a)}{\neg P(a)} \text{ (\wedge ER)}}{P(a) \Rightarrow \perp} \text{ (-E)} \\
\qquad \qquad \qquad \frac{\perp}{P(a) \Rightarrow \perp} \text{ (\Rightarrow E)} \\
\frac{x_0 : \exists x.(P(x) \wedge \neg P(x))}{\frac{\perp}{P(a) \wedge \neg P(a) \Rightarrow \perp} \text{ (\Rightarrow I/x_1)}} \text{ (\exists E)} \\
\frac{\perp}{\exists x.(P(x) \wedge \neg P(x)) \Rightarrow \perp} \text{ (\Rightarrow I/x_0)} \\
\frac{\exists x.(P(x) \wedge \neg P(x)) \Rightarrow \perp}{\neg \exists x.(P(x) \wedge \neg P(x))} \text{ (-I)}
\end{array}$$