Announcements:

- Prelim #1 tonight!
  - Conflict exam: 5:45-7:15 in 315 Upson
    - Only for people with conflicts
  - Main exam: 7:30-9:00 in Goldwin Smith Hall G64
  - Graded (late) tonight, back in section tomorrow
- PS3 due Thursday 11:59PM
  - Testing will start sometime Friday morning
  - Return is likely to be delayed due to Fall break
- Quiz #3 in class Tue Oct 18

Minimal correct induction proof

Example problem you might see on a prelim:

Recall that for any natural number n, we define n! as n(n-1)(n-2)..., where 0! = 1. Write a recursive definition fact n that computes n!, and prove your definition is correct using induction and the substitution model.

Solution:

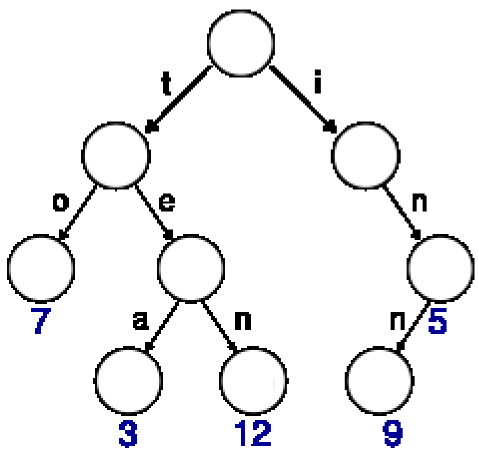
let rec fact(n) = if n=0 then 1 else n*fact(n-1)

* Statement P[n]: the value of the OCaml expression fact(n) is n!
* Variable we are doing induction on: n, starting at 0
* Base case: we prove P[0] as follows

  fact(0)
    b.s.m. (substitute) is
  if 0=0 then 1 else 0*fact(0-1)
    b.s.m. (primitives) is
  if true then 1 else 0*fact(0-1)
    b.s.m. (if) is
  1
  So the value of the expression fact(0) is 1 which is 0!
* Induction step:

  Pick an n >=0 and assume P[n], then prove P[n+1]
  fact(n+1)
    b.s.m. (substitute) is
  if n+1=0 then 1 else n+1*fact(n+1-1)
  Since n >= 0 the value of the expression n+1=0 is false
    b.s.m. (if) is
  n+1*fact(n+1-1)
    b.s.m. (primitives) is
  n+1*fact(n)
  By the induction hypothesis P[n] the value of fact(n) is n! so this is
  n+1*n!
  which is n+1!

- You've seen binary trees in CS2110
- Let's look at a data structure called a "trie"
- A trie is a "finite map", like a dictionary. It maps keys to values. Typically for a trie the keys are strings and the values are numbers.
- A trie is sometimes called a "prefix tree". The basic idea is that a path through the tree represents a prefix, i.e. all strings that start with a particular substring.
    - Root is the empty string
- Example:



- This trie is the finite map `{"to"->7, "tea"->3, "ten"->12, "in"->5, "inn"->9}`
    - As you saw in CS2110, tree-like data structures of this form are very efficient when they are balanced
    - Note that a trie doesn't need to be binary, though this one is
    - In fact, 26 children or so (capitalization, punctuation)
- A trie is very efficient when there are lots of shared prefixes
    - Occurs in many situations (letters, genes, IP addresses)
- Lookup operation is obvious. Insert and delete are surprisingly similar. Everything takes time O(L), which is the length of the longest entry.
- This is a huge advantage of a trie. Most data structures have very asymmetric costs for lookup/insert/delete, so you need to pick the right one for your application carefully.

- Also note that if you don’t find what you are looking for you know something close to it. Useful for, e.g., spell checking.
- Important variant: radix tree (aka Patricia trie), where we ensure that every internal node has 2 or more children by merging nodes with 1 child
- Sub-variant: store at the end “black” or “white”. Then you can use this to encode strings that are present and also strings that are absent. Application is for IP routing tables.

- We will go over the trie signature in section.
- An important idea, both in the trie and point example, is what is called a REP INVARIANT. This is a property of the representation that must be satisfied for the representation to be valid. For example, in our radix tree example, a node must have 2 or more children, and never 1 (could be 0 if it's a leaf).
- You will typically want to implement this with a function repOK that returns its argument or raises an exception.
- Check this on all inputs and on output.
    - This sanity check seems wasteful, and you can turn it off in production code (for example by making repOK into the identity function).
    - But it will catch a ton of subtle bugs
- Example: lists without duplicates, or in sorted order
    - In a certain sense these are types, but they can't be checked at compile time.
    - Another example: even numbers, or prime numbers, or even natural or whole numbers

- But let's now return to the idea of designing a proper specification.
- Deceptively simple example: square root function, float->float
- Spec: beyond the types, what is true before we call sqrt (precondition)
  - What is true after (postcondition)
- What is the actual spec?
  - Positive input
  - Returns "closest" positive float whose square is x
    - Sort of…
- What if the spec is violated?
  - Return something arbitrary? Rarely the right answer
  - Should raise an exception, in general
  - IEEE actually defines an "out of band" value, NaN