

Announcements:

- PS1 due Today 11:59PM
 - Solutions in class tomorrow
 - HW back in section Monday
 - Quiz #1 on Thursday, first 10 minutes of class
 - Coverage includes today's material but not tomorrow's
 - Also returned Monday
 - RDZ office hours: Tuesday after class, but today 4-5, in 4158 Upson
 - Best problem set resource: course staff
 - Best course/exam resource: Yours Truly
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- Need to write the simplest solution to a problem
Important in real life
 - Code that works is simply not good enough
 - “Programs are designed primarily to be read by other humans”
- Important in CS3110
 - Full credit reserved for really the right answer
- Examples:

```
let rec fact(z) =
  if z = 1
  then 1
  else if z = 2
  then 2
  else
    z*fact(z-1)
```

```
let rec inclist (lst: int list) =
  match lst
  with
  | [] -> []
  | [h] -> [h+1]
  | h::t -> h+1::inclist(t)
```

- For CS3110 this is a particularly important lesson because we are going to PROVE code is correct
 - Recall that in ML, as opposed to imperative languages, a program “feels” much more like a mathematical definition

- The main tool used for proofs in CS is mathematical induction
- Today we will do it, briefly, for mathematical formulae
- We will use it for programs in a week or so

- Induction recipe (one of the very few things you should memorize in CS3110):
 - Example: $1+2+\dots+n = n(n+1)/2$
 - 1. Statement to be proven
 - For any natural number n , the sum from 1 to n is $n(n+1)/2$
 - 2. Variable we are doing induction on: n
 - Easy in this case, not always so trivial
 - Call this $P[n]$. Note that it is a sentence about the integer n
 - Not an Ocaml function!
 - 3. Prove base case, typically $P[1]$ or $P[0]$
 - 4. Prove that (for any n) $(P[n] \Rightarrow P[n+1])$
 - Pick an n , assume $P[n]$ is true (I.H.), prove $P[n+1]$ follows
 - Not the same as prove that (for any n) $P[n] \Rightarrow P[n+1]$

- Currying and higher order functions

- Suppose we want to compute $x + \text{sqrt}(y)$

```
let try(x,y) = x +. sqrt(y)
```

- This is short for

```
let try z = match z with (x,y) -> x +. sqrt(y)
```

- Type is $(\text{float} * \text{float}) \rightarrow \text{float}$
- Alternate form, “Currying”, named after logician (not food!)
- Type will be $\text{float} \rightarrow \text{float} \rightarrow \text{float}$
 - What the heck is this?
 - Compare $\text{float} \rightarrow \text{float}$, like `sqrt`
 - Function that takes a float, returns a float
 - Let’s call this a “floatfun”, just to give it a name (slang)
 - Such things are FIRST CLASS OBJECTS
 - Higher order procedures!
- Now we are talking about a function that **returns** a floatfun given a float
 - How to build such a thing in OCaml?

```
let tryc x y = x +. sqrt(y)
```
- This is syntactic sugar for

```
let tryc = fun(x) -> fun(y) -> x +. sqrt(y)
```

 - Which is harder to read
- What is the advantage? Let’s get back to this in a second.

- Simpler example:

```
let plus x y = x + y
```

or with all the types written explicitly:

```
let plus (x : int) (y : int) : int = x + y
```

Notice that there is no comma between the parameters. Similarly, when applying a curried function, we write no comma:

```
plus 2 3 = 2 + 3 = 5
```

The curried declaration above is syntactic sugar for the creation of a **higher-order function**. It stands for:

```
let plus = fun (x : int) -> fun (y : int) -> x + y
```

Evaluation of `plus 2 3` proceeds as follows:

```
plus 2 3
= ((fun (x : int) -> fun (y : int) -> x + y) 2) 3
= (fun (y : int) -> 2 + y) 3
= 2 + 3
= 5
```

So `plus` is really a function that takes in an `int` as an argument, and returns a new function of type `int -> int`. Therefore, the type of `plus` is `int -> (int -> int)`. We can write this simply as `int -> int -> int` because the type operator `->` is right-associative.

It turns out that we can view binary operators like `+` as functions, and they are curried just like `plus`:

```
# (+);;
- : int -> int -> int = <fun>
# (+) 2 3;;
- : int = 5
# let next = (+) 1;;
val next : int -> int = <fun>
# next 7;;
- : int = 8;
```

- So, how does this help us?
- `plus 2` adds 2 to its arg, but without recomputing 2 (so what?)
- How about `plus (slowfun 2)`?