Computability

Announcements:

- PS6 due Monday 12/6 at 11:59PM
  - I will push the course staff about office hours and the newsgroup
- Final exam on Thursday 12/16, 2:00-4:30
  - PS6 tournament and review session M 12/13 and Tu 12/14
  - Which evenings do you prefer?

- What have we covered in CS3110?
- Tools for solving difficult computational problems
  - Abstraction, specification, design
  - Functional programming
  - Concurrency
  - Reasoning about programs
  - Data structures and algorithms

- My personal view of computer scientists versus computer programmers
  - Note that there are 100x as many programmers
- At any time there are some existing programs
- And some programs that don't exist but clearly could
  - Example: problem set (before anyone solves it)
  - Ukrainian spellchecker for Android

- Computer programmers write such programs
- This can be hard work, and well paid
- Always clear that such a program exists,
  - but not necessarily trivial to write it within resource constraints (programmer time, running time/space)

- Computer scientists expand the set of programs we know how to write

- Write programs whose existence is not at all clear
    - Can we make a car that drives itself?
    - Distinguish pictures of cats from dogs?
    - Find broken bones in x-ray images?
    - Create synthetic pictures that look as good as real ones?

- Sometimes we fail
    - Quite often, in fact
    - "If you aren't occasionally failing, then you are working on problems that are too easy."
- Sometimes we discover that a problem is fundamentally hard
    - It wasn't just that the person who tried it wasn't smart enough
- This is the topic of our final lecture

- Boolean-valued functions (true/false) are generally pretty easy to write.
- Consider the following question: does a function of one argument terminate or run forever, given this input?
    - halts(f,a) will be true or false depending on if f(a) halts
    - **Boolean-valued** function
- Note that we aren't going to write in OCaml because types get in the way
- Now consider a new **Boolean-valued** function safely(g)
    - First we check if halts(g,g), and if so we return not(g(g))
    - Otherwise we just return false
    - In pseudocode (NOT in ML) we have

```
safely(g) = if halts(g,g) then not(g,g) else false
```

- Ignoring type checking you can do things like:

```
safely(fun(f)->f(24) != 42)
```

- OK, now what is the value of safely(safely) ?
- It's the value of not(safely(safely)). Oops!

- Resolution: you can't have a function like halts.
- In any language, no matter how smart you are.
- Determining whether or not a program halts is undecidable
- The only way you can figure out what a program does it to run it!
- Related to Cantor's proof of more reals than integers, Goedel's incompleteness proof, Russell's paradox
    - All of these are "diagonalization" arguments

- This has huge real-life consequences.
    - Microsoft design of plug-ins (requiring burglars to sign in)
    - Virtualization
    - Virus issues

- Computer scientists tend to informally say that all programming languages are the same,
    - i.e. anything you can do in one language you can do in another
- There is a mathematically precise way to express this
    - Turing equivalence, see CS3810
    - Taught by John Hopcroft, Turing-award winner
- Weaker languages can actually be better
    - PDF versus postscript

- How do you tell if a problem is undecidable?
- It's not always obvious, though there is one great (sound) heuristic

- Consider the following child's game:
    - We are given types of blocks over symbols, such as a,b,c
    - Infinite set of blocks of each type
    - Find a sequence of blocks so that the top symbols and the bottom ones are the same

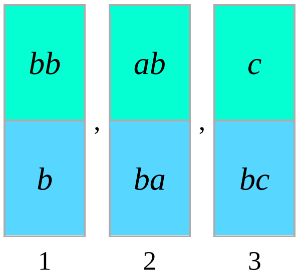
- Example 1:

a		ab		bba
baa	,	aa	,	bb
$i = 1$		$i = 2$		$i = 3$

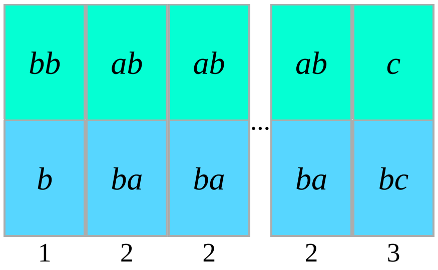
- Solution: 3,2,3,1

bba	ab	bba	a
bb	aa	bb	baa
$i_1 = 3$	$i_2 = 2$	$i_3 = 3$	$i_4 = 1$

- Example 2:



- Solution: 1, any number of 2, 3



- Can we write a program to solve this? It depends!
- For a binary alphabet, it is decidable (first example)
- For an alphabet with 7 or more characters it is undecidable
- For 3 (second example) or more characters it is unknown!

- Suppose we can use no more than k blocks (including copies). Is it decidable?
- Yes – it is finite!
- But it is actually NP-hard, so can’t do better than brute force