CS 2802: Homework 12

November 8, 2020

Handed out Nov. 30, due Dec. 7

• Read Chapter 13.3-13.4 in the handout from Rosen on finite automata (posted on CMS)

• Do the following problems:
  - 13.3, 23
  - 13.3, 25
  - 13.3, 39
  - 13.3, 43 (just state the language in this exercise and the next)
  - 13.3, 45
  - 13.3, 52
  - 13.4, 3(a),(c),(d) (“yes” or “no” suffices here)
  - 13.4, 7(a),(b) (Just write down the regular expression.)
  - 13.4, 23
  - 13.4, 24
  - Additional problem 1: Give an inductive definition of the set of regular expressions (in the spirit of the inductive definition that was given for transitive closure) and show that it is equivalent to the one given in the class notes.
  - Additional problem 2: In the class notes, given a DFA $M_A = (S_A, I, f_A, s_A, F_A)$, I described an automaton $M_A^* = (S_A \cup \{s_0\}, I, f_A^*, s_0, F_A \cup \{s_0\})$, where
    * $s_0$ is a new state, not in $S_A$;  
    * $f_A^*(s, i) = \begin{cases} f_A(s, i) & \text{if } s \in S_A - F_A; \\ f_A(s, i) \cup f_A(s_A, i) & \text{if } s \in F_A; \\ f_A(s_A, i) & \text{if } s = s_0. \end{cases}$

Prove carefully that $M_{A^*}$ accepts $A^*$. (Hint: induction helps for both directions of the proof.)
Challenge problem: Remember the question with the M&M’s on prelim 2? Now suppose that instead of starting with one green candy and \( k - 1 \) red candies, you start with \( m \) green ones and \( k - m \) red ones, where \( m > 0 \) and \( k - m > 0 \). Now what is the probability that the last candy you eat is red? [The solution is really pretty.]

For recitation: 13.3, 25; 13.4, 23; Additional problem 2