CS 2800 today

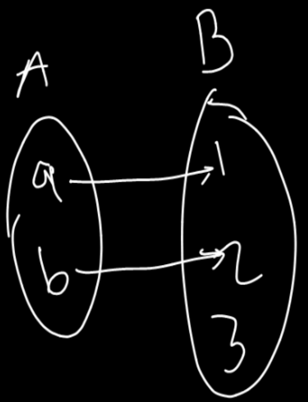
- Cardinality
- Countability

IClicker: I

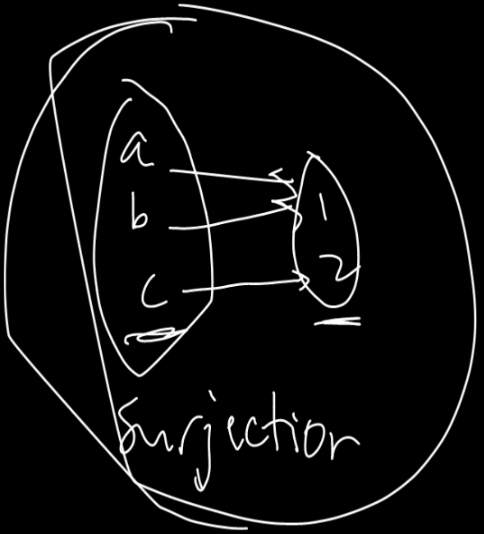
(a) DO

(b) DO NOT

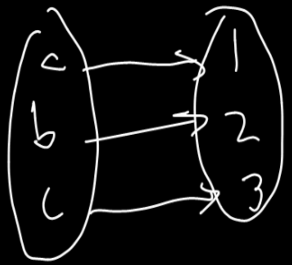
have an I>Clicker



Injective



Surjection



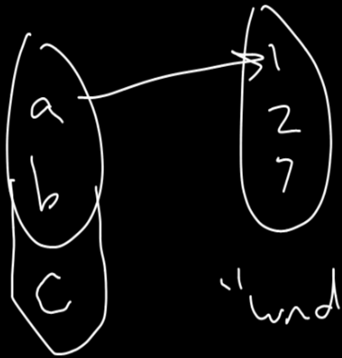
bijection

$(\forall x, y)$ if $f(x) = f(y)$ [and both defined] then $x = y$

idea: if $\exists\, f : A \to B$ injective, then

$$\boxed{|A| \leq |B|}$$

↑ cardinality of A (~~# of elts in A~~)



"undefined"

if A, B are sets, $\boxed{\text{define}}$

$\boxed{|A| \leq |B|}$ as meaning

$$\underbrace{\text{" } \exists \text{ injection } f : A \to B \text{ "}}_{A \text{ inj } B}$$

$\boxed{\text{Def}^{\text{n}}}$: $\boxed{|A| \geq |B|}$ $\boxed{\text{if}}$ there is a surjection $f : A \to B$

if and only if b/c giving a definition

Def^{n} $\boxed{|A| = |B|}$

if $\exists$ $f : A \to B$ which is a bijection.

$|A|$

Things to check

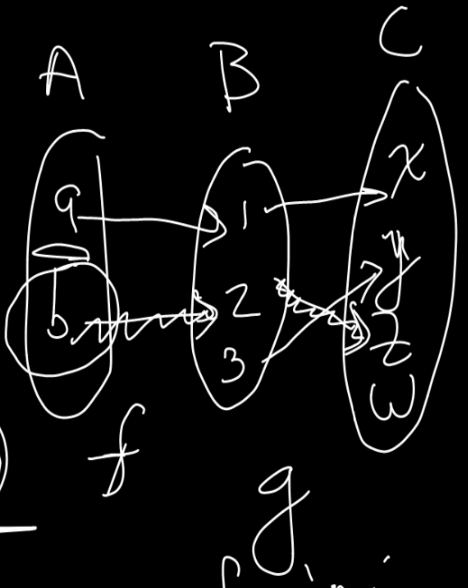
(All true)

- if $|A| \leq |B|$ and $|B| \leq |A|$ then
  $|A| = |B|$
- if $|A| \leq |B|$ then $|B| \geq |A|$
- if $|A| \leq |B|$ then $\boxed{|B| \not< |A|}$?
- if $|A| \leq |B|$ and $|B| \leq |C|$
  then $|A| \leq |C|$

So $\exists f : A \to B$ injective, $\boxed{\exists g : B \to C \text{ (inj)}}$

injective. Know $g \circ f : A \to C$.

I claim $g \circ f$ is an inj. ction.



WTS if $g \circ f(x) = g \circ f(y)$ then $x = y$.

Well, $g \circ f(x) = g(f(x)) = g \circ f(y) = g(f(y))$

since g inj, $f(x) = f(y)$, since f inj,

Know $x = y$

So whenever $(g \circ f)(x) = (g \circ f)(y)$, $x = y$. So $g \circ f$ injective.

So $\exists\, g \circ f : A \to C$ inj, so $|A| \le |C|$

Def^n natural numbers ($\mathbb{N}$) is the set $\{0, 1, 2, \ldots\}$

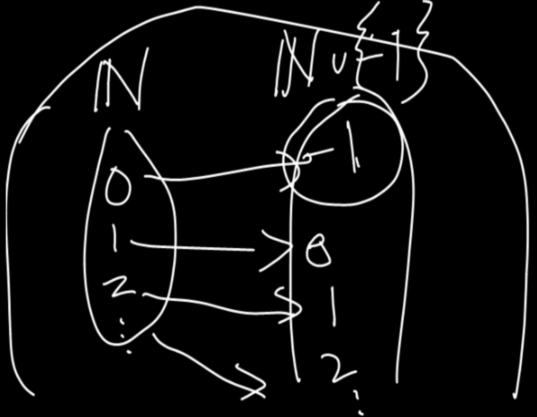
$$|\mathbb{N}| \geq |\underline{\mathbb{N} \cup \{-1\}}|$$

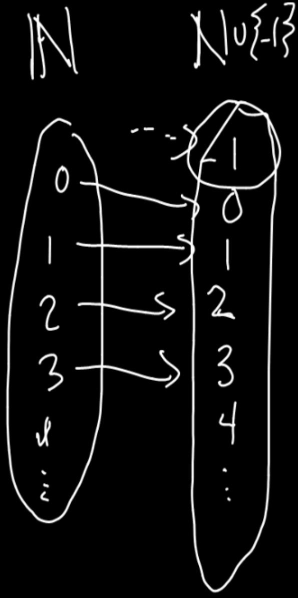
[✓ (A) Yes]

(B) No

(C) Not sure

$n \mapsto n-1$



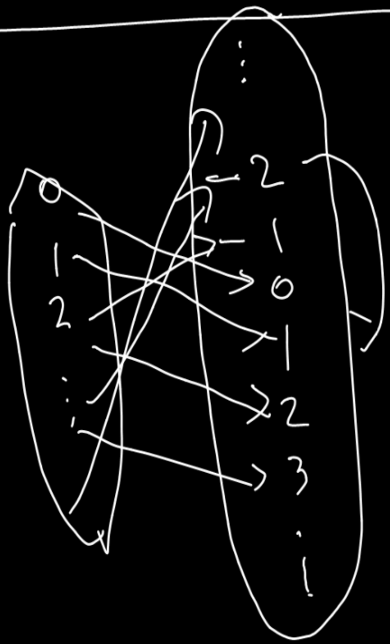


not surj

nothing maps to -1

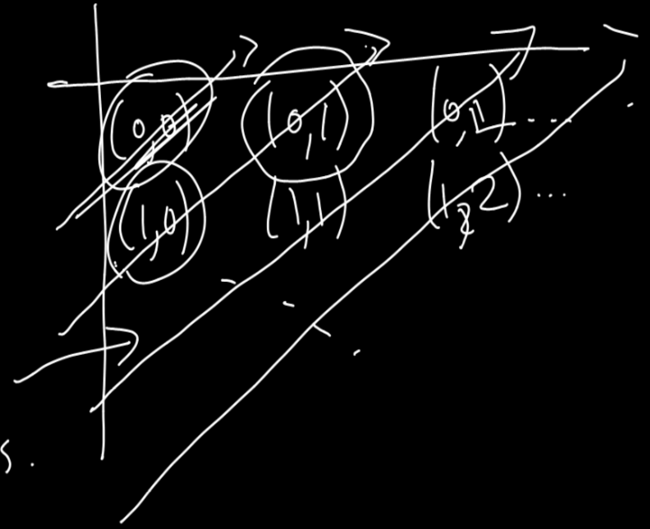
Countable Sets

- $\mathbb{N} \cup \{-1\}$
- Even naturals
- Set of pairs of $\mathbb{N}$ $(\mathbb{N} \times \mathbb{N})$ is countable



$0 \leftarrow 0$
$1 \leftarrow 1$
$-1 \leftarrow 2$
$2 \leftarrow 3$
$-2 \quad 4$
$3 \quad 5$
-3
$\vdots$

$0 \rightarrow x_0$
$1 \rightarrow x_1$
$2 \rightarrow x_2$
3
$\vdots$



pairs.

every pair hit by f.
(f is surjective)

$|\mathbb{N}| \geq |\mathbb{N} \times \mathbb{N}|$

$\mathbb{N}$	$\xrightarrow{f}$ $\mathbb{N} \times \mathbb{N}$
0	(0,0)
1	(1,0)
2	(0,1)
3	(2,0)
4	(1,1)
5	(0,2)
	⋮

Def

"f is an injection"

does not imply

f is not a bijection.

2800 today

- an uncountable set
- start induction,

Defn: X is countable if $\underline{|\mathbb{N}| \geq |X|}$, i.e. if $\exists$ surjection $f: \mathbb{N} \to X$

Facts:

$f: \mathbb{N} \to \mathbb{N} \cup \{-1\}$

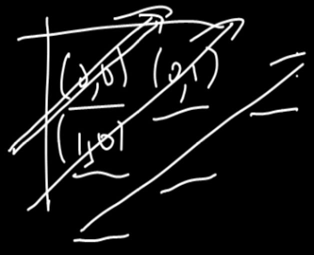
$f(n) ::= n - 1$

$\underline{\mathbb{N} \cup \{-1\}}$ is countable

$\underline{\mathbb{N} \times \mathbb{N}}$ is countable

(review)

f

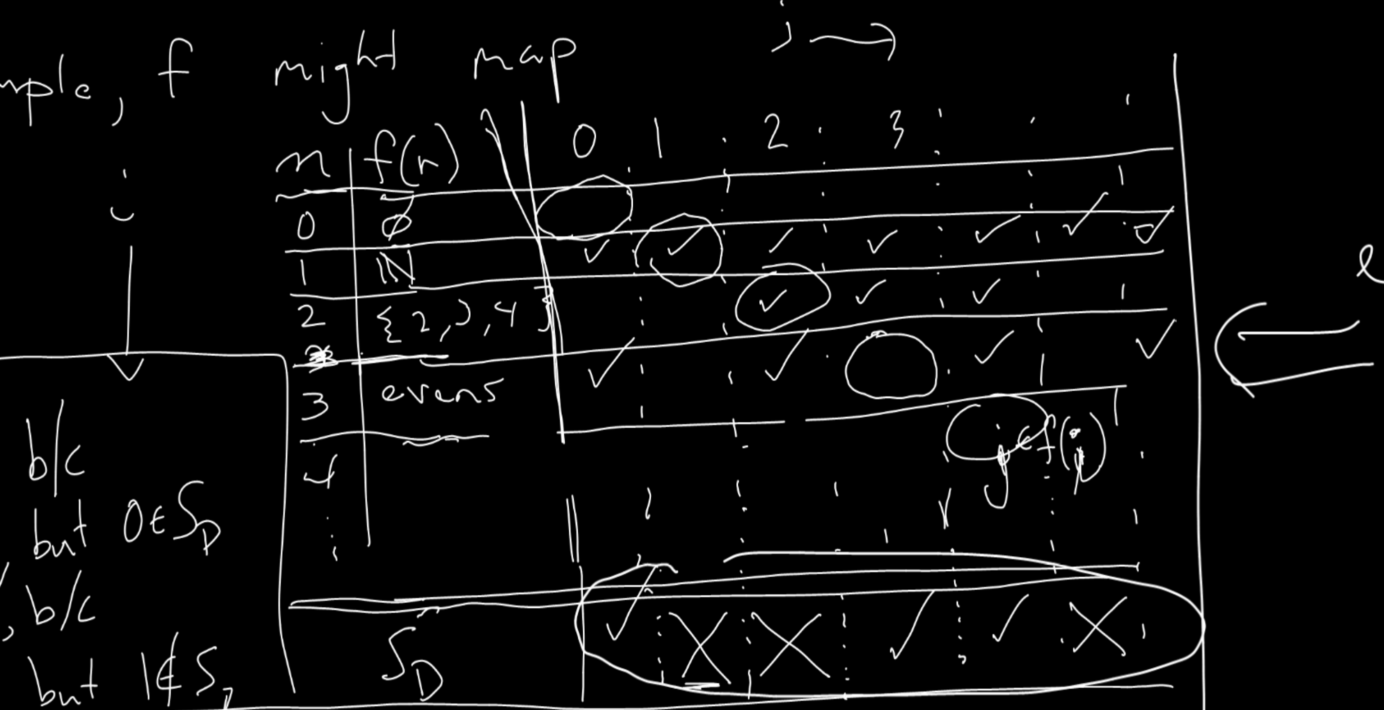


rable

ntradiction. Assume pow(N) is countable.

$\exists$ surj. $f : \mathbb{N} \to pow(\mathbb{N})$.

mple, f might map



$S_D = \{ i \in \mathbb{N} \mid i \notin f(i) \}$. Then S_D differs

$f(i)$ in i^{th} column, so $S_D \neq f(i$

diction because f is surjective.

Claim: $\mathbb{R}$

n	$f(n$
0	0.12
1	0.11
2	0.12
3	0.00

x_D 0.2

Diagonal