CS 2800 today

- a bit more proofs
- functions and
Last time, \( N \) is a function satisfying

rule 0 \( \forall i, j, j' : \text{ if } \& (i, j) = f(i, j') \),

rule 2 \( \forall i, i', j, j' : \text{ if } \& (i, j) = f(i', j) \),

rule 3 \( \forall i, i', j, j' : \text{ if } f(i, j) = f(i', j') \text{ then } i = i' \text{ or } j = j' \).

Claim: \( f(9, 3) = 1 \).

Proof: I must be \( f(i, j) \) for some \( i, j \).

\( f(4, 1) = 1 \) (rule 2). Since \( j \neq 2 \).
Claim: Each number $k \in \{1, \ldots, 9\}$ appears in each row, also in each column, each square, and there is a box which does not appear in row i. Then remaining entries in row i (of there are 9) can only be elements of the
Functions

MG functions

\{ \text{total: every input gives an output, partial function: may not give an output for some inputs}\}
Claim: $g(5) \text{ contains a 1}$

$$f(8, 5) = 1$$

Proof: assume $f(8, 5) = 1$.

since 1 has to be in lower-left-hand corner, it can't be in row 2, b/c it could be in row 3.
Claim: If \( x^2 + 3x + 2 = 0 \)

\[ x = -1 \]

Proof: \( x = -2 \)

\[ (-1)^2 + 3(-1) + 2 = 0 \]

\[ 1 - 3 + 2 = 0 \]
Notation: \( f : A \to B \) means \( f \) is a function, domain is \( A \), codomain is \( B \).

Set of possible inputs

Set of possible outputs
not a (total) function

is a partial function

\[ f : A \to B \]

is a function.

colonelm is \( B \).

\[ f(x) := x^2 \]
Def: $f: A \rightarrow B$ is injective (an injection) if whenever $f(x_1) = f(x_2)$ for all $x_1, x_2 \in A$ then $x_1 = x_2$.

Because 2 els map to same output.

Not an injection.
Def. \( f : A \rightarrow B \) is a surjection if for all \( y \in B \), \( \exists x \in A \) such that \( f(x) = y \).
really nice funs: bijects, functions that are both injective and surjective.

Nutrition: if \( f \) is surjection then \( B \) is "bigger" then \( A \).

\[ f : A \rightarrow B \text{ then } f \text{ is injection} \]