CS 2800 today

- a bit more proofs
- functions and

$N = \{1, 2, \ldots 9\}$

$j \rightarrow$

1	2	3	4	5	6	7	8	9	
	1			5			7		1
	3	7	1		2	6	8		2
1		6				5		3	3
			5		9				4
			2		1				5
5		3		7		1		4	6
		8				7			7
	4						9		8
									9

$N =$

Last time:

Sudoku soln is a function satisfying

rule ① $\forall i, j, j'$, if $f(i,j) = f(i,j')$

rule ② $\forall i, i', j$, if $f(i,j) = f(i'$

rule ③ $\forall i, i', j, j'$, if $f(i,j) = f(i',j')$, square(i,j) = square(i',j') then i

Claim: $f(9,3) = 1$.

Proof: 1 must be $f(i,j)$ for some i,j

$f(4,1) = 1$ (rule 2). Sim. $j \neq 2$

Claim (Lemma): each ~~row~~ number $k \in \{1 \ldots 9\}$ a

each row, also in each col,

each square

there e

Suppose $\exists$ i, k such

k allows not appear in row i.

then remaining entries in row i (of

there are 9) not k can only be elements of the

will the 9 slots ...

functions

MCS functions
- total: every input gives an output
- partial function: may not give an output for some inputs.

				8				
	1			5			7	
	3	7	1		2	6	8	
1		6				5		3
			5		9			
			2		1			
5		3		7		1		4
		8				7		
	4	1					9	

Claim: if f is a sud. soln c
$f(8,5)$ contains a 1

$f(8,5)=1$.

Proof: $\boxed{\text{assume } f(8,5)=1}$ T
since 1 has to af
in lower-left-hand s
it can't be in row 8, b/c
assumption can't be in row

Claim: if $x^2+3x+2=0$

$x=-1$ or $x=-2$

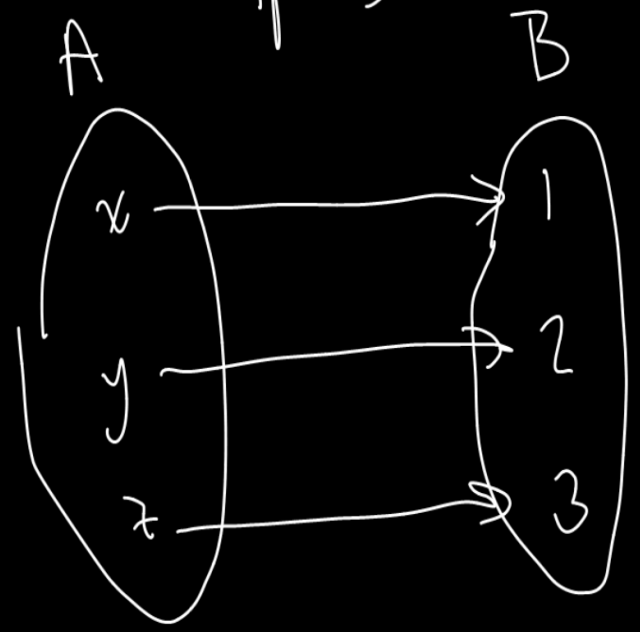
Proof: $(-1)^2+3(-1)+2=0$ Bagh

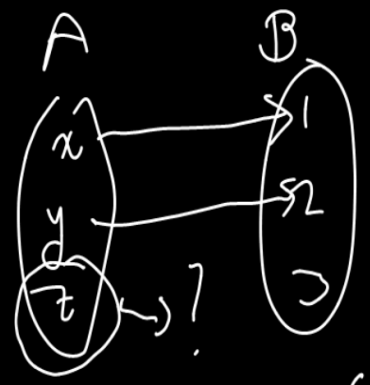
$1-3+2=0$

Notation: $f : A \to B$ means f is a function, **domain** is A, **Codomain** is B.

set of possible inputs

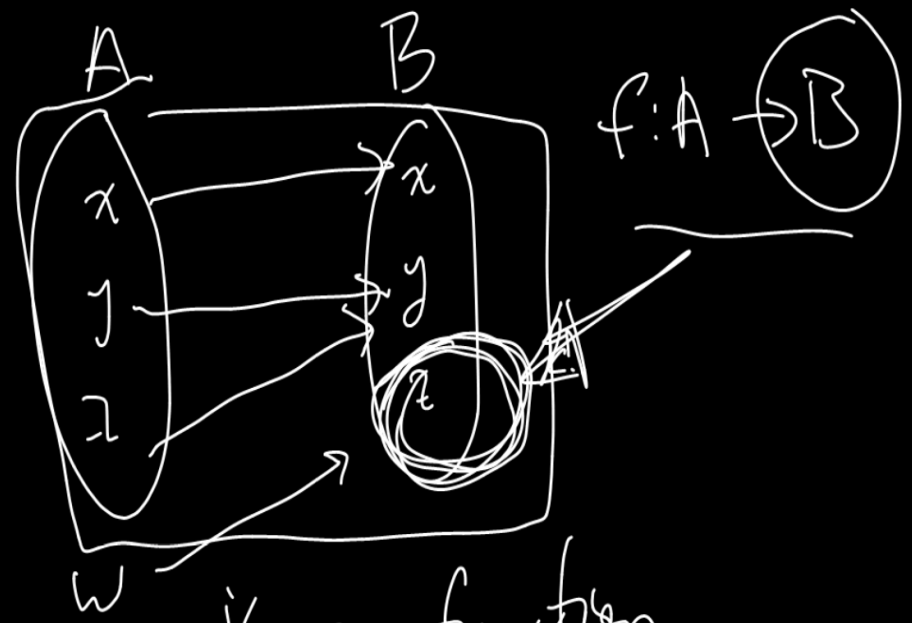
Set of possible outputs





not a (total) function

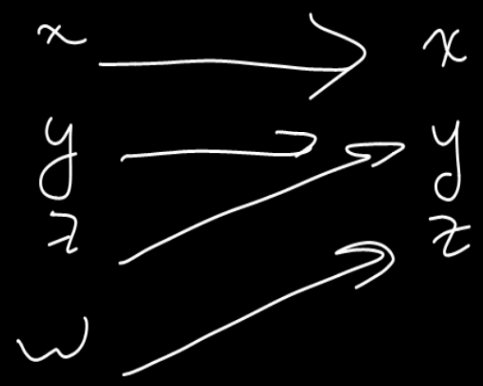
is a partial function



$f : A \to B$

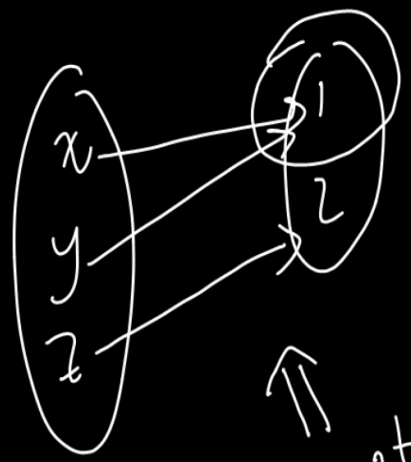
is a function.

codomain is B.



$f(x) := x^2$

$x_1 \neq x_2$



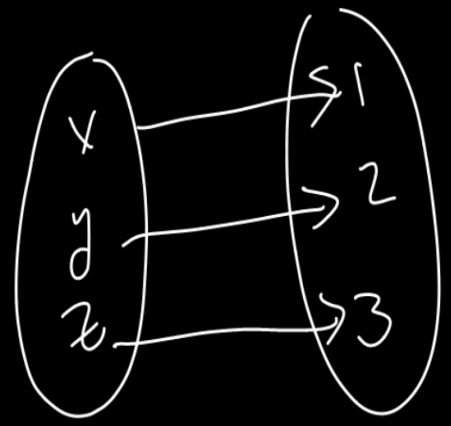
$\Uparrow$

not an injection

x_1

x_2

$x_1 = x_2$

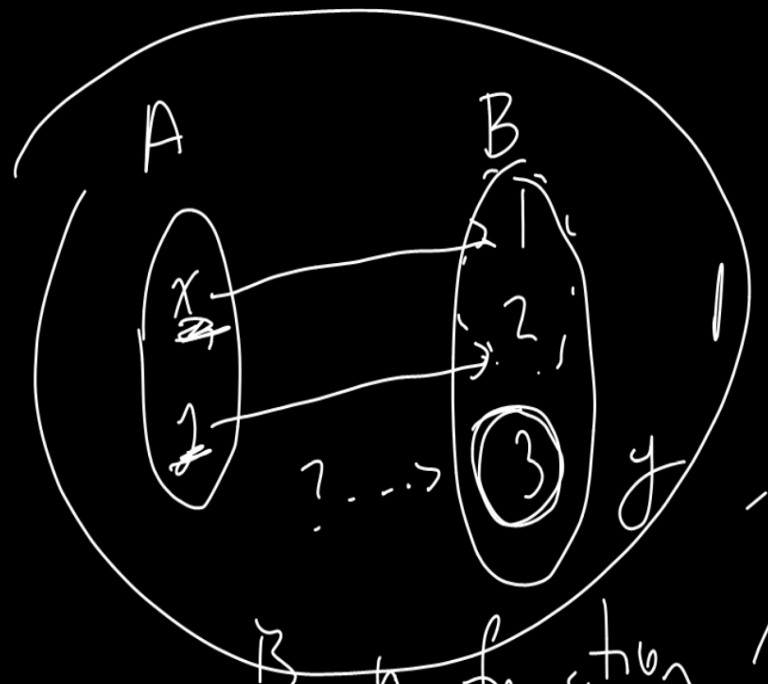


because 2 elts map to same output.

Def$^{\underline{n}}$: $f: A \to B$ is **injective** (an injection)

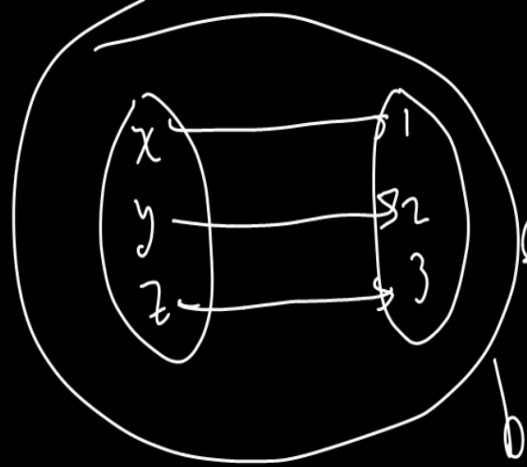
if whenever (for all $x_1, x_2 \in A$) $f(x_1) = f(x_2)$,

$x_1 = x_2$.



B is a function

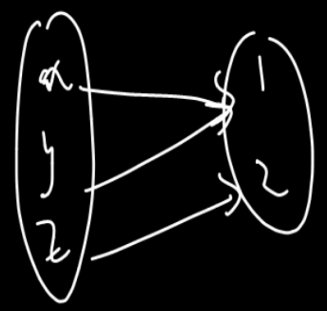
not a surjection



is a surjection

because every output in codomain gets hit.

Def^n $f: A \to B$ is a <u>surjection</u> if for all $y \in B$, $\exists\, x \in A$ such that $f(x) = y$.

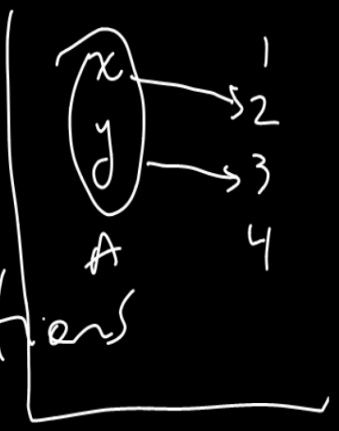


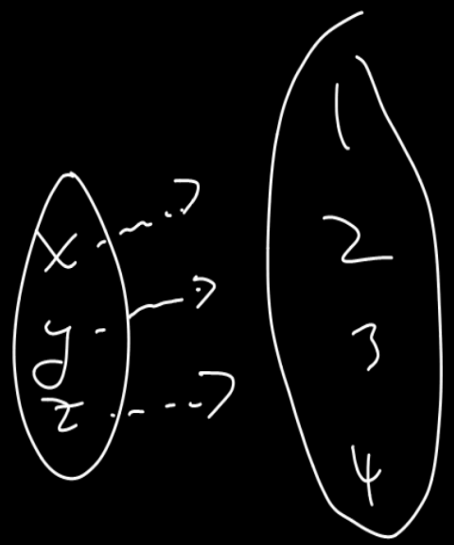
is a surjection

not injective.

really nice funs:

bijections, functions that are both injective <u>and</u> surjective.





<u>intuition</u>

if $\exists$ surjection $f: A \to B$ then B is "smaller" than A.

if $\exists$ injection $f: A \to B$ then B is "bigger" than A.