Finite Automata

CS 2800: Discrete Structures, Spring 2015

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A simplified model

String → Machine → String
A simplified model

String → Machine → String

Finite sequence of letters from an alphabet $\Sigma$, e.g. $\{0, 1\}$
A general-purpose computer

String → Turing Machine → String
A general-purpose computer

String → Turing Machine → String

we might revisit these later
A general-purpose computer

Church-Turing Thesis: Any “effective/mechanical/real-world” calculation can be carried out on a Turing machine
A simple “computer”

String$\rightarrow$Deterministic Finite Automaton (DFA)$\rightarrow$Yes/No
An example
An example

(Binary input)
An example

Input: 01001
An example

Input: 01001
An example

Input: 01001
An example

Input: 01001
An example

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Input: 01001
```
An example

Input: 01001
An example

Input: 01001

Yes

No
Input: 01001
Output: Yes!
An example

In general, on what binary strings does this DFA return Yes?
An example

Ans: All strings with an even number of 1's
Deterministic Finite Automaton

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  - $q_0 \in Q$ is the start/initial state
  - $F \subseteq Q$ is the set of final/accepting states
What does this DFA accept?

$q_0$

(any symbol)
What does this DFA accept?

Answer: No strings
What does this DFA accept?

\[ q_0 \]

(any symbol)
What does this DFA accept?

Answer: All strings
What does this DFA accept?
What does this DFA accept?

Answer: Strings of length 1
What does this DFA accept?

Answer: Strings of length 1
What does this DFA accept?
What does this DFA accept?

Answer: Strings containing only 1's
What does this DFA accept?
What does this DFA accept?

Answer: Strings containing no two consecutive 1's
Language

- Given alphabet $\Sigma$, a language $L$ is a set of strings over the alphabet, i.e. $L \subseteq \Sigma^*$
Language

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• We say a language $L$ is accepted/recognized by a DFA $M$, if $M$ accepts input string $x \in \Sigma^*$ if and only if $x \in L$
Language

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Set of all possible strings over $\Sigma$
Language

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What language does this DFA accept?
What language does this DFA accept?

Answer: Only the string 1
What language does this DFA accept?
What language does this DFA accept?

Answer: Only the string 11
DFA's find it difficult to count

- A DFA that recognizes the language $\{1^c\}$ (the single string of $c$ 1's) must have at least $c$ states
DFA's find it difficult to count

• A DFA that recognizes the language \( \{1^c\} \) (the single string of \( c \) 1's) must have at least \( c \) states
  – The parent alphabet is irrelevant (but must of course contain 1)

(Proof discussion to be completed next class)