Lecture 5

Sets not countably infinite

functions mapping \{1, 2, 3, \ldots\} \rightarrow \{0, 1\}

The class of all sets of integers

The diagonal function.

If \(f_1, f_2, f_3, \ldots\) is a list of functions, then \(f_D(i) = f_i(i) + 1 \mod 2\) is not on the list.

Can diagonalize over a list of computer programs and hence over all computable partial functions.

\[
f_D(i) = \begin{cases} 
p_i(i) + 1 & \text{if } p_i(i) \text{ halts} \\
0 & \text{otherwise}
\end{cases}
\]

is a non computable function.

Halting problem is not computable. Otherwise \(f_D(i)\) would be computable.

There exists a theorem that is true but not provable. Otherwise we could solve the halting problem.