Proofs

CS 2800: Discrete Structures, Fall 2014

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Thought for the Previous Day

Is this statement true or false?

“All eleven-legged alligators have orange and blue spots”

Space for boardwork
Thought for the Previous Day

This statement is *true***!!!

... else there would be an eleven-legged alligator (which lacks orange spots, or blue spots, or both)

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Recap: left and right inverses

- $g : B \rightarrow A$ is a **left inverse** of $f : A \rightarrow B$ if $g(f(a)) = a$ for all $a \in A$

- $h : B \rightarrow A$ is a **right inverse** of $f : A \rightarrow B$ if $f(h(b)) = b$ for all $b \in B$
Recap: left and right inverses

- A function is *injective* (one-to-one) iff it has a *left inverse*
- A function is *surjective* (onto) iff it has a *right inverse*
Thought for the Day #1

If a function has both a left inverse and a right inverse, then the two inverses are identical, and this common inverse is unique.

Bijection and two-sided inverse

- Two-sided inverse of $f: A \rightarrow B$ is a function $g: B \rightarrow A$ that is both a left inverse and a right inverse
- A function $f$ is bijective iff it has a two-sided inverse

Proof worked out on board here, also see http://www.cs.cornell.edu/courses/cs2800/2014fa/handouts/jonpak_function_notes.pdf
A proof from set theory

For any two sets $A$ and $B$,

$$A = (A \cap B) \cup (A \setminus B)$$

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Definition: Set equality

\[ A = B \text{ if and only if } A \subseteq B \text{ and } B \subseteq A \]
Definitions: Set operators

- $A \cup B$ consists of all elements in $A$ or $B$ (or both!)
- $A \cap B$ consists of all elements in both $A$ and $B$
- $A \setminus B$ consists of all elements in $A$ but not in $B$
- $A'$ consists of all elements not in $A$
A proof from set theory

For any two sets $A$ and $B$,

$$A = (A \cap B) \cup (A \setminus B)$$
Another proof from set theory

For any two sets $A$ and $B$,

$$A \setminus B = A \cap B'$$

Prove this on your own!
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I carved and carved, and the next thing I knew I had two pumpkins.

I told you not to take the axiom of choice.
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Let $a = b = 1 \ldots$