1. (3 points) A broken proof: A student is asked to prove that $17 = 35$. They submit the following “proof”:

\[
\begin{align*}
17 & = 35 \\
17 - 26 & = 35 - 26 \\
(17 - 26)^2 & = (35 - 26)^2 \\
17^2 - 2 \times 26 \times 17 + 26^2 & = 35^2 - 2 \times 26 \times 35 + 26^2 \\
17^2 & = 35^2 - 2 \times 26 \times 18 \\
289 & = 1225 - 936 \\
289 & = 289
\end{align*}
\]

(subtract 26 from both sides)
(square both sides)
(expand)
(cancel $26^2$)
(add $2 \times 26 \times 17$ to both sides)
(plug it into a calculator)
(Q.E.D.)

Explain the faulty reasoning in this “proof”.

2. Prove from first principles (set theory, Kolmogorov’s axioms) or give a counterexample for each of the following:
   
   a) (3 points) $P(A \cap B) \leq P(A)$ for any events $A$ and $B$.
   
   b) (3 points) If $A \subseteq B$ but $A \neq B$ then $P(A) < P(B)$.
   
   c) (3 points) If $A_1, A_2, \ldots, A_n$ are mutually exclusive events and $\bigcup_i A_i = S$ (the entire sample space), then for some $i$, $P(A_i) \geq 1/n$.

3. (3 points) Two Cornell students missed their final exam because they were partying in New York City the night before. Desperate for a make-up test, they lied to the professor that they had a flat tire while returning. The professor agreed to give them a make-up test, as long as the students sat in separate rooms. When they opened the paper, they found a single question, worth 100 points: “Which tire was it?”

What’s the probability the two students will give the same answer? Justify your result and clearly state any assumptions you made.

4. Drawing pairs:
   
   a) (2 points) You have 10 red, 10 green and 10 blue pairs of socks in a drawer. What’s the probability that if you randomly pull out two socks without looking, they will be the same color?

   b) (3 points) You have 10 red, 10 green and 10 blue pairs of shoes in a (very large) drawer. What’s the probability that if you randomly pull out two shoes without looking, they will be the same color and a left-right pair?

Justify your answer in both cases.