CS 2800: Discrete Structures

Homework 3

Due Monday, September 17, 2012

[Version 2 - Updated Tuesday, September 11, 2011]

Please write your netid on the upper right corner of all pages. Grading for all problems will be based on neatness, style, and correctness.

1. Consider the non negative integers \( N = \{0, 1, 2, \ldots \} \), and define \( i \equiv j \) to be whether the remainder of \( i \) divided by 5 equals the remainder of \( j \) divided by 5.

(a) Is the \( \equiv \) relation reflexive? Is it symmetric? Is it transitive?

(b) What are the equivalence classes?

(c) Select the smallest integer in each equivalence class to represent the equivalence class. Let \( \{i\} \mod \) be the representative for the class containing \( i \). Prove the following statements:

i) \( \{i\} \mod + \{j\} \mod \equiv \{i + j\} \mod \)

ii) \( \{i\} \mod \times \{j\} \mod \equiv \{i \times j\} \mod \).

(d) Which of the following are valid statements? Explain.

i. \( 5 \equiv 10 \)

ii. \( 6 \equiv 7 \)

(e) Write out the addition and multiplication tables for the representative elements of the equivalence classes so that \( \{i\} \mod + \{j\} \mod \equiv \{i + j\} \mod \) and \( \{i\} \mod \times \{j\} \mod \equiv \{i \times j\} \mod \).

(f) How can one add negative numbers to this system? That is, replace \( N \) with \( Z \).

2. (a) List three groups besides \((Z, +)\).

(b) Give an example of a non commutative group.

(c) How do you prove that \( \{I, r, r^2, r^3, f, fr, fr^2, fr^3\} \) is a group under concatenation?

3. Prove that between every two rationals there exists a real.

4. Prove that between every two reals there exists a rational.

5. Let \( R \) be the reals. Is there a one-to-one mapping from \( R^2 \) to \( R \)?

6. A finite game is a game with a finite number of steps and no ties. Does every finite game have a winning strategy for either player 1 or player 2? Justify your answer.