Discrete Structures	**Problem Set 2**
CS 280 Spring 2005	**Due February 9, 2005**

Reading: Rosen Section Section 3.2.

(1) For each of the following English sentences, write a sentence that is equivalent to its negation. You answer should not be of the form: "It is not true that..." or anything equivalent.

For example, suppose the sentence is: "All students in CS280 this semester have taken CS100 last semester." Then the negation would be: "Some student in CS280 this semester was not taking CS100 last semester."

You may want to first write the original sentence as "$\forall$ students $x \in$ (this semester's CS280), student x took CS100 last semester." Then negate it: "$\exists$ student $x \in$ (this semester's CS280) such that x did not take CS100 last semester." Then convert to English: "Some student in CS280 this semester was not taking CS100 last semester." But all you have to hand in, is the last sentence.

Notes. (1) You are not asked to decide if the statements are true or false, just negate them. (2) Some English sentences are ambiguous. If you feel that this is the case with the sentences below, explain what are the different meanings, and which one you are using for the problem.

(a) For every query it receives, Google responds in 2 seconds.

(b) Microsoft has sold at least 75% of all personal computer software in each country in the word.

(c) For every legislation that any senator votes against, at least 5 other senators also opposed.

(d) Every CS major at Cornell knows a biology major.

(e) Every point in Florida is within 50 miles of a beach.

(f) Every point in Florida is within 20 miles of a beach or a city.

(2) In the following question you are asked to decide if some implications are true. For example, if we assume "All giraffes are tall.", and "All tall animals eat grass." This implies that "All giraffes eat grass.". However, adding the assumption that "No mouse is tall." Does not imply that "No mouse eats grass". Say which of the implications are true. For those that are **not** true, also give a brief explanation.

As in the previous question: if you find the sentences ambiguous, explain the possible different meanings, and say which one you are using. Also, you do not have to evaluate if the assumptions are true, only decide if they imply the conclusion.

(a) Assumptions:
All furry animals eat meet.
Giraffes do not eat meet.

Proposed conclusion:
Giraffes are not furry.

(b) Assumptions:
Many airlines had financial problems in the last 5 years.
Airlines that are in financial trouble cancel flights.
USAir is an airline.

Proposed conclusion:
USAir has canceled flights in the last 5 years.

(c) Assumptions:
No small city has both a *Borders* and a *Barns and Noble* Store.
Ithaca is a small city.

Proposed conclusion:
Ithaca has neither *Borders* nor *Barns and Noble.*

(3) Problem deferred till next week.

(4) For each natural number $n \geq 1$ define

$$f(n) = \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n(n+1)}.$$

Prove that $f(n) = 1 - \frac{1}{n+1}$ for all $n \geq 1$.

(5) Has been deferred till next week.

(6) Recall the definition of $n!$, which is $1! = 1$, and for $n \geq 1$ we define $(n+1)! = (n+1) \cdot n!$. We want to understand how fast $n!$ grows. Clearly, we have that $n! \leq n^n$, as we get $n!$ by multiplying together n numbers, but how fast does it grow, that is we want to know what is the lower bound on the value of $n!$?

To answer this, we need a fact from calculus, that you may use without proving it.

Fact. For the number $e \approx 2.718...$ and all $n \geq 1$ we have

$$\left(1 + \frac{1}{n}\right)^n < e.$$

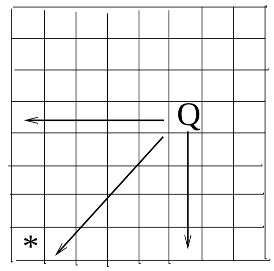

Figure 1: The Board Game in Problem 8.

Prove that for all natural numbers $n \geq 1$

$$n! \geq \left(\frac{n}{e}\right)^n.$$

(7) Consider a two player game, where there are n coins on the table to start, the players alternate turns taking coins away. At each turn a player may take 1 or 2 coins. The player who takes the last coin wins.

(a) In class we considered the tree of all possible states of the game. Draw the game tree for $n = 5$.

In this game different partial plays can result in the same game position: For example, starting with n coins, if the first player takes 2 and then the second player takes 1, then the first player needs to move with $n - 3$ chips left. You get the same position if the first player takes 1, and the second takes 2. You may use a single node for such equivalent partial plays, but you don't have to.

(b) Prove that the first player has a winning strategy if and only of n is not divisible by 3.

(c) Consider the version of this game with n chips where a player on his turn can take away up to k chips (rather than just 2 as was defined originally).

Decide who has a winning strategy as a function of n and k. Prove that your answer is correct.

(8 optional) Consider a game played on the chess board of 8×8 squares with a single queen. It starts with the queen in some position on the board. The queen moves in its usual way, except that it may only move south, west, or south-west. See the figure for a position, where the queen is indicated by a Q, and the lines indicate the squares where the queen can move to in one step. Two players alternate steps, and the player who gets the queen in the south-west corner (marked by a * on the figure) wins. Which are the positions from which if started, player one can force a win (that is player one has a winning strategy). Prove your answer.