Review Questions for COM S 280 final.
(The question numbers correspond to fifth edition of the book)

Chapter 5 (Review Question 11)

a) What does it mean to say that a random variable has a geometric distribution with prob-
ability p?

b) What is the mean of a geometric distribution with probability p?

Chapter 5 (Supplementary Exercise 28)

Use Chebyshev’s Inequality to show that the probability that more than 10 people get the cor-
rect hat back when a hatcheck person returns hats at random does not exceed 1/100 no matter
how many people check their hats. (Hint: Use Example 6 and Exercise 35 in Section 5.3) [Please
look at these examples and exercises from the book]

Chapter 5 (Supplementary Exercise 31)

There are n different types of collectible cards you can get as prized when you buy a particular
product. Suppose that every time you buy this product it is equally likely that you get any
these of these cards. Let X be the random variable equal to the number of products that need
to be purchased to obtain at least one of each type of card and let $X_j$ be the random variable
equal to the number of additional products that must be purchased after $j$ different cards have
been collected until a new card is obtained for $j = 0, 1, 2, ..., n - 1$.

a) Show that $X = \sum_{j=0}^{n-1} X_j$.

b) Show that after $j$ distinct types of cards have been obtained, the card obtained with the
next purchase will be a card of a new type with probability $\frac{n-j}{n}$.

c) Show that $X_j$ has a geometric distribution with parameter $\frac{n-j}{n}$.

d) Use parts (a) and (c) to show that $E(X) = n \sum_{j=1}^{n} \frac{1}{j}$.

e) Use the approximation $\sum_{j=1}^{n} \frac{1}{j} \approx \ln(n) + \gamma$ where $\gamma = 0.57721...$ is Euler’s constant, to
find the expected number of products that you need to buy to get one card of each type
if there are 50 types of cards.

Section 5.3 (Exercise 30)

Suppose that the number of cans of soda pop filled in a day at a bottling plant is a random vari-
able with an expected value of 10000 and a variance of 1000.

a) Use Markov’s Inequality to obtain an upper bound on the probability that the plant will
fill more than 11000 can on a particular day.

b) Use Chebyshev’s Inequality to obtain a lower bound on the probability that the plant will
fill between 9000 and 11000 can on a particular day.

Section 8.4 (Exercise 14)

Show that all vertices visited in a directed path connecting two vertices in the same strongly
connected component of a directed graph are also in this strongly connected component.

Section 9.4 (Exercise 21)

Describe the trees produced by a breadth-first search and a depth-first search of the complete
bipartite graph $K_{m,n}$ starting at a vertex of degree $m$, where $m$ and $n$ are positive integers.
Justify your answers.

Section 9.4 (Exercise 42)

Use Exercise 39 to prove that if $G$ is a connected, simple graph with $n$ vertices and $G$ does not
contain a simple path of length $k$ then it contains at most $(k - 1)n$ edges.