Review Questions for COM S 280 final.

(The question numbers correspond to fifth edtion of the book)

Chapter 5 (Review Question 11)

a) What does it mean to say that a random variable has a geometric distribution with probability p?

b) What is the mean of a geometric distribution with probability p?

Chapter 5 (Supplementary Exercise 28)

Use Chebyshev's Inequality to show that the probability that more than 10 people get the correct hat back when a hatcheck person returns hats at random does not exceed $1/100$ no matter how many people check their hats. (*Hint*: Use Example 6 and Exercise 35 in Section 5.3) [Please look at these examples and exercises from the book]

Chapter 5 (Supplementary Exercise 31)

There are n different types of collectible cards you can get as prized when you buy a particular product. Suppose that every time you buy this product it is equally likely that you get any these of these cards. Let X be the random variable equal to the number of products that need to be purchased to obtain at-least one of each type of card and let X_j be the random variable equal to the number of additional products that must be purchased after j different cards have been collected until a new card is obtained for $j = 0, 1, 2, ..., n - 1$.

a) Show that $X = \sum_{j=0}^{n-1} X_j$.

b) Show that after j distinct types of cards have been obtained, the card obtained with the nest purchase will be a card of a new type with probability $\frac{n-j}{n}$.

c) Show that X_j has a geometric distribution with parameter $\frac{n-j}{n}$.

d) Use parts (a) and (c) to show that $E(X) = n\sum_{j=1}^{n} \frac{1}{j}$.

e) Use the approximation $\sum_{j=1}^{n} \frac{1}{j} \approx \ln(n) + \gamma$ where $\gamma = 0.57721...$ is Euler's constant, to find the expected number of products that you need to buy to get one card of each type if there are 50 types of cards.

Section 5.3 (Exercise 30)

Suppose that the number of cans of soda pop filled in a day at a bottling plant is a random variable with an expected value of 10000 and a variance of 1000.

a) Use Markov's Inequality to obtain an upper bound on the probability that the plant will fill more than 11000 can on a particular day.

b) Use Chebyshev's Inequality to obtain a lower bound on the probability that the plant will fill between 9000 and 11000 can on a particular day.

Section 8.4 (Exercise 14)

Show that all vertices visited in a direccted path connecting two vertices in the same strongly connected component of a directed graph are also in this strongly connected component.

Section 9.4 (Exercise 21)

Describe the trees produced by a breadth-first sarch and a depth-first search of the complete bipartite graph $K_{m,n}$, starting at a vertex of degree m, where m and n are positive integers. Justify your answers.

Section 9.4 (Exercise 42)

Use Exercise 39 to prove that if G is a connected, simple graph with n vertices and G foes not contain a simple path of length k then it contains at most $(k-1)n$ edges.