Ten Powerful Ideas

- **Counting**: Count without counting (*combinatorics*)
- **Induction**: Recognize it in all its guises.
- **Exemplification**: Find a sense in which you can try out a problem or solution on small examples.
- **Abstraction**: Abstract away the inessential features of a problem.
  - One possible way: represent it as a graph
- **Modularity**: Decompose a complex problem into simpler subproblems.
- **Representation**: Understand the relationships between different possible representations of the same information or idea.
  - Graphs vs. matrices vs. relations
- **Refinement**: The best solutions come from a process of repeatedly refining and inventing alternative solutions.
- **Toolbox**: Build up your vocabulary of abstract structures.

- **Optimization**: Understand which improvements are worth it.
- **Probabilistic methods**: Flipping a coin can be surprisingly helpful!

Connections: Random Graphs

Suppose we have a random graph with n vertices. How likely is it to be connected?

- What is a *random* graph?
  - If it has n vertices, there are $C(n,2)$ possible edges, and $2^{C(n,2)}$ possible graphs. What fraction of them is connected?
  - One way of thinking about this. Build a graph using a random process, that puts each edge in with probability $1/2$.

- Given three vertices a, b, and c, what's the probability that there is an edge between a and b and between b and c? $1/4$
- What is the probability that there is no path of length 2 between a and c? $(3/4)^{n-2}$
- What is the probability that there is a path of length 2 between a and c? $1-(3/4)^{n-2}$
- What is the probability that there is a path of length 2 between a and every other vertex? $> (1-(3/4)^{n-2})^{n-1}$

Now use the binomial theorem to compute $(1-(3/4)^{n-2})^{n-1}$

$$\begin{aligned} & (1-(3/4)^{n-2})^{n-1} \\ = \; & 1-(n-1)(3/4)^{n-2}+C(n-1,2)(3/4)^{2(n-2)}+\cdots \end{aligned}$$

For sufficiently large n, this will be (just about) 1.

Bottom line: If n is large, then it is almost certain that a random graph will be connected.

Theorem: [Fagin, 1976] If P is *any* property expressible in first-order logic, it is either true in almost all graphs, or false in almost all graphs.

This is called a *0-1 law*.

Connection: First-order Logic

Suppose you wanted to query a database. How do you do it?

Modern database query language date back to SQL (structured query language), and are all based on first-order logic.

- The idea goes back to Ted Codd, who invented the notion of relational databases.

Suppose you're a travel agent and want to query the airline database about whether there are flights from Ithaca to Santa Fe.

- How are cities and flights between them represented?
- How do we form this query?

You're actually asking whether there is a path from Ithaca to Santa Fe in the graph.

- This fact cannot be expressed in first-order logic!

Some Bureuacracy

- The final is on Thursday, Dec. 16, 12-2:30 PM, in Olin 255
- If you have conflicts (more than two exams in a 24-hour time period) let me know as soon as possible.
  - Also tell me the courses and professors involved (with emails)
  - Also tell the other professors
  - There will probably be a makeup on Dec. 16.
- Office hours go on as usual during study week, but check the course web site soon.
  - I'll be out of town Dec. 6
  - There may be small changes to accommodate the TAs exams
- There will be two review sessions,
  - Tuesday, Dec. 14, 7-8:30, Upson 205
  - Wednesday, Dec. 15, 7-8:30, Upson 205

Coverage of Final

- everything covered by the first prelim
  - emphasis on more recent material
- Chapter 4: Fundamental Counting Methods
  - Basic methods: sum rule, product rule, division rule
  - Permutations and combinations
  - Combinatorial identities (know basic theorems covered in notes)
  - Pascal's triangle
  - Binomial Theorem (but not multinomial theorem)
  - Balls and urns
  - Inclusion-exclusion
  - Pigeonhole principle
- Chapter 6: Probability:
  - 6.1–6.7
  - basic definitions: probability space, events
  - conditional probability, independence, Bayes Thm.
  - random variables

  - uniform, binomial, and Poisson distributions
  - expected value and variance
  - Markov + Chebyshev inequalities
  - understanding Law of Large Numbers, Central Limit Theorem
- Chapter 7: Logic:
  - 7.1–7.4, 7.6-7.8; *not* 7.5
  - translating from English to propositional (or first-order) logic
  - truth tables
  - algorithm verification
  - first-order logic