ADJACENCY MATRIX $G = (V, E)$ - SIMPLE GRAPH

$A_G = [a_{ij}] \quad 1 \leq i, j \leq |V|$

\[a_{ij} = \begin{cases} 
1, & \text{if } \{v_i, v_j\} \in E \\
0, & \text{o.w.}
\end{cases}\]

EXAMPLE

\[
\begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\]

$A_{ii} = 0$ since $G$ has no loops

$a_{ij} = a_{ji}$ since $\{v_i, v_j\} = \{v_j, v_i\}$
Matrices for Multi- (Pseudo-) Graphs

\[
\begin{bmatrix}
1 & 2 & 1 \\
2 & 3 & 1 \\
1 & 1 & 0
\end{bmatrix}
\]

Matrices for Directed Graphs

\[
\begin{bmatrix}
1 & 1 & 1 \\
2 & 2 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

Not necessarily symmetric matrix

Incidence Matrices

\[ G = (V, E) \]

\[ M_G = \begin{bmatrix}
\vdots & \vdots & \vdots \\
v_1 & v_2 & \ldots v_n \\
\vdots & \vdots & \vdots
\end{bmatrix}
\]

\[ m_{ij} = \begin{cases} 
1, & \text{if } e_j \text{ is incident with } v_i \\
0, & \text{otherwise.}
\end{cases} \]

Two ones in each column

Also works for all other sorts of graphs

\[ G = \begin{bmatrix}
e_1 & e_2 & e_3 \\
v_1 & v_2 & v_3
\end{bmatrix}
\]

\[ \begin{bmatrix}
1 & 0 & 1 \\
1 & 1 & 0 \\
0 & 1 & 1
\end{bmatrix} = M \]

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ISOMORPHISM OF GRAPHS

\[ G_1 = (V_1, E_1); \quad G_2 = (V_2, E_2) \]

Simple graphs are isomorphic if there is a bijection \( f: V_1 \rightarrow V_2 \) such that \( \{a, b\} \in E_1 \iff \{f(a), f(b)\} \in E_2 \).

Invariants of isomorphisms:

- # of vertices
- # of edges
- Degrees of vertices
- Connectivity
- etc.

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Example. \(K_5\) \hspace{1cm} \(W_4\)

Not isomorphic: 10 edges in \(K_5\) vs. 8 edges in \(W_4\)

Not isomorphic: one has a vertex of degree 1...

Ah. A bijection \(f: V_1 \rightarrow V_2\)

is an isomorphism of \((V_1, E_1)\) and \((V_2, E_2)\)

iff the adjacency matrix of \((V_1, E_1)\)

coincides with the adjacency matrix of \((f(V_1), E_2)\)
EXAMPLE.

DETERMINE WHETHER THE GIVEN PAIR OF GRAPHS IS ISOMORPHIC.

\[ \begin{array}{c}
\text{deg} = 4 \\
\end{array} \]