RECURSIVE SEQUENCES

Example. The size of a certain fish population in Cayuse lake can increase 10% a year due to natural growth. The harvesting rate is 100 individuals per year. If the initial population size is 8000 individuals find the population size after 5 years.

\[ x_n = \text{the population size after } n \text{ years.} \]
\[ x_0 = 8000; \quad x_{n+1} = 1.1 \cdot x_n - 1000 \]

Initial Condition

Recurrence Relation

\[ x_1 = 1.1 \cdot x_0 - 1000 = 8800 - 1000 = 7800 \]
\[ x_2 = 1.1 \cdot x_1 - 1000 = 7580 \]
\[ x_3 = 1.1 \cdot x_2 - 1000 = 7338 \]
\[ x_4 = 1.1 \cdot x_3 - 1000 = 7072 \]
\[ x_5 = 1.1 \cdot x_4 - 1000 = 6779 \]

Solution of the Recurrence Relation (Sequence)

-141-

Compound Interest. The initial deposit is $1000 at a bank yielding 5% per year with interest compounded annually. How much will be the amount after 7 years?

\[ S_0 = 1000 \]
\[ S_1 = 1.05 \cdot S_0 \]
\[ S_2 = 1.05 \cdot S_1 = 1.05 \cdot 1050 \]
\[ S_3 = 1.05 \cdot S_2 = 1.05 \cdot 1102.50 \]
\[ S_4 = 16288.95 \]
\[ S_5 = 16288.95 \]

-142-

Example. Messages are transmitted through a communications channel using two signals. One requires 1 microsecond, the other 2.

\[ a_n = \text{# of different messages that can be send in } n \text{ microseconds. (No blanks)} \]
\[ a_1 = 1, \quad a_2 = 1. \] Each signal of the length \( n \) is obtained by one and only one of the following two procedures:

1. \[ = n \]

2. \[ = n \]

\[ a_n = a_{n-1} + a_{n-2}; \quad a_0 = 1, \quad a_1 = 1 \]
\[ 1, 1, 2, 3, 5, 8, 13, \ldots \]

Parenthesizing products.

\[ x_0 \cdot x_1 \cdot x_2 \quad (x_0 \cdot x_1) \cdot x_2 \]
\[ x_0 \cdot (x_1 \cdot x_2) \quad (x_0 \cdot x_1) \cdot x_2 \]
\[ (x_0 \cdot x_1) \cdot x_2 \quad (x_0 \cdot x_1) \cdot x_2 \]

Catalan Numbers

\[ C_1 = 1, \quad C_2 = 2, \quad C_3 = 5, \ldots \]

\[ C_n = C_{n-1} + C_{n-2}; \quad C_0 = 0, \quad C_1 = 1 \]

-143-