CS280    Fall 2001    Prelim 1

Full name:                      Student ID:

Statement of integrity: I did not, and will not, break the rules of academic integrity on this exam.

(Signature)

You must **show your work** and/or give clear explanations or reasons for each answer you give. If you use a formula, show all parts of the formula before simplifying. Correct answers without explanation will be worth 0. Also, please indicate clearly your final answer (e.g. by drawing a box around it or similarly highlighting it).

1. (4 points each) Determine which of the following propositions are tautologies:
   a) \((p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)\)

   b) \(((p \rightarrow q) \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))\)  
   (Pay attention to the parentheses!)

   c) \((s \rightarrow p) \lor (\neg p \land s)\)
2. (4 points each) Determine whether each of the following statements is true or false. The domain of discourse in each case is the real numbers.

a) $\exists x \exists y ((x > y) \rightarrow (x^2 < y^2))$

b) $\forall x \exists y \forall z ((x < y) \rightarrow ((z > x) \land (z < y)))$

c) $\forall x (((x > 0) \rightarrow (x^3 < 0)) \lor ((x^3 < 0) \rightarrow (x > 0)))$

3. (5 points each)

a) Let $A = \{\emptyset, \{\emptyset\}\}$. Find $P(A)$, the power set of $A$. 
b) Show that \((A - C) \cap (C - B) = \emptyset\). You must give a logical argument, not just show Venn diagrams.

4. (10 points) Let \(X\) and \(Y\) be sets and suppose there is an injection (1-to-1 function) \(f : X \to Y\) (but \(f\) is not necessarily onto). Prove that there is a surjection (onto function) \(g : Y \to X\) (i.e., for each \(x \in X\), there exists \(y \in Y\) so that \(g(y) = x\)).
5. (5 points each)
   a) Find \( \sum_{n=1}^{200} n \)

   b) Find \( \sum_{n=0}^{25} 3(-3)^n \)

6. a) (2 points each) Which of the following have solutions? Explain why or why not. If there is a solution, use the Euclidean algorithm to find it.
   i) \( 8x \equiv 1 \mod 12 \)

   ii) \( 7x \equiv 1 \mod 30 \)

   iii) \( 100x \equiv 1 \mod 102 \)
b) (5 points) If the product of two integers is $2^73^85^27^{11}$ and their greatest common divisor is $2^33^45$, what is their least common multiple? Explain.

7. (10 points) Prove by induction that 5 divides $11^n - 6$ for all positive integers $n$. 