For some of the solutions, we are only providing the answer. This is for your benefit: it is much faster for us to type up the answer than it is to show all of the steps. Thus, you get this solution set much faster.

1 Truth Tables

The most straightforward solution is \((\neg p \land q \land \neg r) \lor (p \land \neg q \land r) \lor (p \land q \land \neg r)\).

2 Existential Quantification

(Note there are many possible solutions to this one.)

Let the domain for variable \(x\) be \(\{1, 2\}\), \(A(x)\) be the proposition “\(x = 2\)”, \(B(x)\) be the proposition “\(x = 0\)”. Then \(A(1)\) is false, and \(B(1)\) is true, so \(A(1) \rightarrow B(1)\) is true; therefore \(\exists x[A(x) \rightarrow B(x)]\) is true. However, \(A(2)\) is true, and \(\exists xB(x)\) is false. Therefore \(\exists xA(x) \rightarrow \exists xB(x)\) is false.

3 Empty sets relating to sets

(a) \(\emptyset \in \{\{\emptyset\}\}\) No
(b) \(\emptyset \subseteq \{\{\emptyset\}\}\) Yes
(c) \(\{\emptyset\} \in \{\{\emptyset\}\}\) Yes
(d) \(\{\emptyset\} \subseteq \{\{\emptyset\}\}\) No- The subsets of \(\{\{\emptyset\}\}\) are \(\emptyset\) and \(\{\emptyset\}\).
(e) \(\{\emptyset\} \in P(\{\{\emptyset\}\}\) No- equivalent to (d).

4 1-1 functions

Let \(f : S \rightarrow T\) and \(g : T \rightarrow U\) be functions.

(a) **Claim:** If \(g \circ f\) is 1-1, then \(f\) is 1-1.

**Proof.** Given \(x, y \in S\), suppose \(f(x) = f(y)\). Then \(g(f(x)) = g(f(y))\) since \(g\) is a well-defined function, i.e. \((g \circ f)(x) = (g \circ f)(y)\). Since \((g \circ f)\) is 1-1, \(x = y\). We conclude that \(f\) is also 1-1.

(b) (Note there are many possible solutions to this one.)

Let \(S = T = U = R\), the set of real numbers. Define \(g(x) = x^2\), and \(f(x) = e^{x/2}\). \(g(x)\) is not 1-1: for example, \(g(1) = g(-1) = 1\). However, \((g \circ f)(x) = e^x\) is 1-1.

5 Matrices

The answer is \(
\begin{pmatrix}
0 & 0 \\
0 & 0 \\
0 & 0
\end{pmatrix}
\).

\(A \cdot B + A \cdot C + A \cdot D = A \cdot (B + C + D)\). And it is straightforward to see that \(B + C + D\) is null.
6 Fermat’s Little Theorem

\[ 49 = 7^{1000002} \mod 101. \]

7 Chinese Remainder Theorem

Using the theorem, \( x = 1002. \)

8 Induction

Induction Hypothesis:

\[
\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}
\]

Base case: \( n = 1. \) Clearly, \( \frac{1}{(2-1)(2+1)} = \frac{1}{2+1}. \)

Induction step: Assume the induction hypothesis is true for \( n. \) We prove it is true for \( n+1. \)

\[
\sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)} = \sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2(n+1) - 1)(2(n+1) + 1)}
\]

\[
= \frac{n}{2n+1} + \frac{1}{(2(n+1) - 1)(2(n+1) + 1)} \quad \text{by induction hypothesis}
\]

\[
= \frac{n}{2n+1} + \frac{1}{(2n+1)(2n+3)} = \frac{n(2n+3)}{(2n+1)(2n+3)} + \frac{1}{(2n+1)(2n+3)} = \frac{n(2n+3) + 1}{(2n+1)(2n+3)}
\]

\[
= \frac{2n^2 + 3n + 1}{(2n+1)(2n+3)} = \frac{(2n+1)(n+1)}{(2n+1)(2n+3)} = \frac{(n+1)}{2(n+1) + 1}. \]

By induction, the hypothesis is true for any positive integer \( n. \)