1. (15 points) Determine which of the following propositions are tautologies
   a) \((p \rightarrow \neg p) \leftrightarrow \neg p\)  
   b) \((p \rightarrow \neg q) \leftrightarrow \neg(p \land q)\)  
   c) \(((\neg p \land q) \rightarrow r) \rightarrow ((\neg q \rightarrow p) \rightarrow r)\).

2. (15 points)
   a) Establish the logical equivalence of 
      \(\forall x (A \rightarrow B)\) and \(\exists x (A \land \neg B)\).
   b) Show that \(\exists x (A(x) \land B(x))\) and \(\exists x A(x) \land \exists x B(x)\) are not logically equivalent.

3. (15 points) The composition of functions \(f\) and \(g\), denoted by \(f \circ g\), is defined by \((f \circ g)(a) = f(g(a))\). The inverse of \(h\) is the function \(h^{-1}\) such that \(h^{-1} \circ h\) and \(h \circ h^{-1}\) are identity functions, i.e. \((h^{-1} \circ h)(a) = a\) and \((h \circ h^{-1})(b) = b\) for all \(a\) from the domain of \(h\) and all \(b\) from the codomain of \(h\).
   a) Give an example of \(f\) and \(g\) such that \(f \circ g\) and \(g \circ f\) are different.
   b) Suppose \(f\) and \(g\) are invertible. Show that \((f \circ g)^{-1}\) equals to \(g^{-1} \circ f^{-1}\).

4. (10 points)
   a) How many multiplications does the standard row-column algorithm uses to compute the product of an \(m \times n\) matrix and an \(n \times p\) matrix? Explain why.
   b) Suppose you have to find \(A \cdot B \cdot C\), were \(A\) is a \(3 \times 10\) matrix, \(B \cdot 10 \times 50\) matrix and \(C - 50 \times 2\) matrix. Which order should you choose: \((A \cdot B) \cdot C\) or \(A \cdot (B \cdot C)\)?

5. (15 points) Compute the greatest common divisor (gcd) of 156 and 93. Find integers \(x\) and \(y\) such that \(156x + 93y = \text{gcd}(156, 93)\).

6. (10 points)
   a) Find the base 8 expansion of \((123)_{10}\).
   b) Find the binary expansion of \((123)_{10}\)

7. (20 points) By the Chinese Remainder Theorem for each integers \(a, b\) and \(c\) (0 ≤ \(a\) < 9, 0 ≤ \(b\) < 10 and 0 ≤ \(c\) < 11) there is a unique nonnegative integer \(x < 990 = 9 \cdot 10 \cdot 11\) such that \(x \equiv a\, (\text{mod} 9)\), \(x \equiv b\, (\text{mod} 10)\) and \(x \equiv c\, (\text{mod} 11)\).
   a) Find such \(a\), \(b\) and \(c\) for \(x = 801\).
   b) Find a positive integer \(x\) satisfying \(x \equiv 1\, (\text{mod} 9)\), \(x \equiv 0\, (\text{mod} 10)\) and \(x \equiv 1\, (\text{mod} 11)\).