Handed out: Mon., Apr. 9.

Due: Wed., Apr. 18 in lecture.

The policies for this (and other problem sets) are as follows:

- You should hand in your on-time problem set in lecture in the box at the front of the room on the day it is due. Problem sets handed in elsewhere (TA’s office, Upson 303, etc.) will be considered late. See the next bullet.

- Late papers may be handed in up to 24 hours late. For instance, this problem set may be handed in up to 11:00 am on Apr. 19. You can hand in a late paper in Upson 303. Late papers get an automatic deduction of 10%. The full late penalty is applied even if you turn in part of the solution on time.

- Problem sets may be done individually or in teams of two. Put your name or names on the front page. Re-read the academic integrity statement on the web for the policy concerning working in larger groups.

- Problem sets count for 20% of the final course grade. The lowest scoring problem set will be dropped.

- You need Matlab for some of the questions. Matlab is available in the following CIT labs: Upson and Carpenter.

- If you need clarification for a homework question, please either ask your question in section, lecture, or office hours or else post it to the newsgroup cornell.class.cs222. The professor reads this newsgroup and will post an answer.

- Write your section number (like this: “Section 2”) at the top of the front page of your paper, and circle it. This is the section where your graded paper will be returned. As a reminder, Section 1 is Th 12:20, Section 2 is Th 3:35, Section 3 is F 2:30, Section 4 is F 3:35.

1. Section 8.1.1 of the text describes the “divide-and-average” iteration for finding square roots. By carrying out some algebra, show that this iteration is equivalent to a Newton method.

   [Hint: To get started, you should write down a nonlinear equation $f(x)$ whose coefficients involve $A$ and whose root is $\sqrt{A}$. Then write down Newton’s formula for your nonlinear equation to see if you can obtain, after some algebra, the divide-and-average method. There is more than one way to write down the nonlinear equation, and not all of them work.]

2. The function $f(x) = \arctan(x)$ in radians has a single root at $x = 0$ and provides an interesting test case for Newton’s method. Try to determine using some combination
of graphing, analysis, and Matlab (the function is \texttt{atan} in Matlab), for which starting points Newton’s method will converge to the root of arctan($x$) and for which starting points it will diverge. Hand in your answer along with any relevant plots or Matlab tests.

3. Recall that the points and weights for Gaussian quadrature are determined by solving a system of nonlinear equations. Consider the three-point Gauss quadrature rule for the interval $[-1, 1]$. There are six parameters in this rule, namely $w_1, w_2, w_3, x_1, x_2, x_3$. Write down (on paper) the six nonlinear equations for these six parameters. (Review section 4.4.1 in the text if you don’t know how to write these equations.) Also, write down (on paper) the Jacobian of the nonlinear system.

Then implement (in Matlab) a Newton method for solving for these parameters. Terminate when the residual (i.e., $\|f(x^*)\|_2$) is below $10^{-15}$. Compare the solution you get to the known 3-point quadrature rule. (You can find the rules in many textbooks, in the course textbook’s website, or on several other website that can be found with a search engine.)

Hand in: the written material listed above, your m-file(s) to solve for the six parameters. Try two different starting points:

$$[w_1^0, w_2^0, w_3^0, x_1^0, x_2^0, x_3^0] = [2/3, 2/3, 2/3, -1, 0, 1]$$

and

$$[w_1^0, w_2^0, w_3^0, x_1^0, x_2^0, x_3^0] = [5, 6, 7, 8, 9, 10].$$

Make your m-file print out of the value of $w_2$ computed on each step of the Newton iteration. (Note: use \texttt{format long} to make all digits visible.) Also, print out all entries of the solution to which Newton’s method converges, assuming the program terminates.