CS 222: Introduction to Scientific Computing
Spring 2001
Prelim 2

Handed out: Thursday, April 12, 7:30 pm.

PLEASE DO NOT OPEN THIS BOOKLET UNTIL THE SIGNAL IS GIVEN. This examination lasts 75 minutes and has 75 points total. It is closed book and closed note, but you are permitted to use an $8\frac{1}{2}'' \times 11''$ crib-sheet with notes written on both sides. This exam counts for 25% of your final course grade. There are 8 questions total spread over pages 2–4 of this booklet. Write your answers in this booklet. You can separate the pages since we have staplers available at the front of the room. You can write continuations on the last page if one of your answers is too long to fit in the space.

Name:

first name(s)  last name(s)

CU ID number:

NetID:

Section #

Note: Section 1 is Thurs, 12:20, section 2 is Thurs, 3:35, section 3 is F 2:30 and section 4 is F 3:35.

“Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use or receive unauthorized aid in this examination.”

Sign in the box →
1. **[5 points]** Is the $2 \times 2$ matrix 
\[
\begin{pmatrix}
.9 & .1 \\
-.1 & .9
\end{pmatrix}
\]
a Givens rotation? Why or why not?  
**Answer here:**

2. **[5 points]** Let $C$ be a symmetric positive definite matrix whose Cholesky factorization is $C = GG^T$. What kind of matrix is $G$? Be as specific as possible.  
**Answer here:**

3. **[5 points]** Consider finding a root of $f(x) = \sin x - x^2 - .5$ using Newton’s method. Write down the formula for $x^{(k+1)}$ in terms of $x^{(k)}$ for this particular $f$.  
**Answer here:**

4. **[5 points]** Write down an example of a $2 \times 2$ matrix that is simultaneously symmetric and orthogonal.  
**Answer here:**
5. **[10 points]** Consider finding roots of the function \( f(x) = x^2 - 9 \). Which of the following intervals would work best as the starting interval if you were planning to use the bisection method: \([-2, 0]\), \([0, 2]\), or \([2, 4]\)? Briefly explain why.

**Answer here:**

6. **[15 points]** Let \( U \) be an \((n + 1) \times n\) matrix that is upper triangular except for the last row. (In other words, \( U(i, j) = 0 \) for all \((i, j)\) satisfying \( j < i \leq n \).) How many flops, accurate to the leading term, are required to reduce this matrix to upper triangular form using Givens rotations?

**Answer here:**
7. **[15 points]** For the matrix $U$ in the last question, suppose one wants to solve $\min \|Ux-b\|_2$ for some vector $b$ using the method of normal equations. Determine how many flops (accurate to the leading term) are required for this computation.

**Answer here:**

8. **[15 points]** Let $U$ be an $n \times n$ matrix that is upper triangular except for a single entry in the lower triangular portion at position $(i, j)$ (so $i > j$). Can this matrix be reduced to upper triangular with a single Givens rotation? (In other words, is there an $n \times n$ Givens rotation $G$ such that $GU$ is upper triangular?) Explain your answer. Most likely your answer will depend on $i$ and $j$.

**Answer here:**
Write continuations to your answers on this page. Label the continuations.

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