The exam is closed book and closed notes. You have 150 minutes. Good luck!

Write your name and Cornell NetID at the top of every page! There are 6 questions on 10 numbered pages, front and back. Check that you have all the pages. When you hand in your exam, make sure your booklet is still stapled together. If not, please use our stapler to reattach all your pages!

Scrap paper is available. If you do a lot of crossing out and rewriting, you might want to write code on scrap paper first and then copy it to the exam, so that we can make sense of what you handed in.

Write your answers in the space provided. Ambiguous answers will be considered incorrect. You should be able to fit your answers easily into the space provided.

In some places, we have abbreviated or condensed code to reduce the number of pages that must be printed for the exam. In others, code has been obfuscated to make the problem more difficult. This does not mean that it’s good style.

**Academic Integrity Statement:** I pledge that I have neither given nor received any unauthorized aid on this exam.

______________________________
(signature)
1. Short Questions (20 points)

(a) Topological sort (4 points)

Topological sort of a DAG can be written abstractly as shown below, putting the nodes in topological order into an ArrayList. State the condition under which the topological ordering is unique, i.e. there exists only one topological ordering.

Set<Node> g1= a copy of the nodes of graph g (so changing g1 will not change g);
ArrayList<Node> res= new ArrayList<Node>();
while (g1.size() > 0) {
    Let n be a node of g1 with indegree 0;
    res.add(n);
    Remove node n and all edges connected to it from g1;
}

(b) Tree traversal (4 points)

Show that you cannot uniquely construct a binary tree from its preorder and postorder traversals by drawing two different trees that have the same preorder and postorder traversals. Hint: Make the trees as small as possible.

(c) Hashing with linear probing (4 points)

Consider hashing with linear probing using an array b[0..6] to maintain a set of integers. Do not be concerned with the load factor. The hash function to be used is just the integer itself: hashcode(i) = i. Draw array b after these values have been added: 11, 13, 20, 14.
(d) Comparable (4 points)

Function `compareTo`, which is defined in interface `Comparable`, need not return −1, 0, or 1 but can return any integer. This is seen in the specification of `compareTo` in class `Pt`, given below.

(1) Fill in whatever implements clause is required in the class definition below. (2) Complete the two return statements in the body of `compareTo` below. Do not use conditional expressions.

```java
/** An instance represents a point in the plane.
   * Points are ordered by x-values, e.g. (5, 8) comes before (6, 3), but on
   * y-values if the x-values are the same, e.g. (5, 3) comes before (5, 4). */
public class Pt {
    public int x;
    public int y;

    /** Return a negative number, 0, or positive number depending on whether
     * this point comes before p, is the same as p, or comes after p. */
    public int compareTo(Pt p) {
        if (x != p.x) return ;
        return ;
    }
    ...
}
```

(e) Exception handling (4 points)

Below is a scheme for a try-statement, with two catch clauses. `S1`, `S2`, and `S3`, represent sequences of statements. Explain how this try-statement is executed.

```java
try { S1 }
catch (ArithmeticException e) { S2 }
catch (RuntimeException e) { S3 }
```
2. Object-Oriented Programming (18 points)

You and your pals maintain online collections of pics (pictures), each of which is tagged with information like the date and the people in the pictures. Egotistical YOU would like to search through your pals’ pictures, their pals’ pictures, etc., and get hold of all pictures that have you in them.

You will write a program to get those pics in three steps. Step (b) will use the method from step (a), and step (c) uses (b).

Notes: Just take one method at a time, read its specification carefully, and implement it. The only loops you need to write are for-each loops. Here are two classes that you will use.

```java
/** An instance represents a person, their pals, and their pics. */
public class Person {
    public String name;
    public Set<String> tags;
    public Set<Person> pals;
    public Set<Pic> pics;
}

/** An instance is a picture with tags. */
public class Pic {
    public JPG image; // you won’t need this
    public Set<String> tags;
}
```

(a) Gathering who’s pics (6 points) Write the body of the following procedure:

```java
/** Add to setPics all of p’s pics that have tag who and are not already in setPics. */
public static void gatherPics(Person p, String who, Set<Pic> setPics) {
}
```

(b) Recursive gather (6 points) Write method `gather`, below, using method `gatherPics` of part (a).

```java
/** If p is not in set seen, then:
 * (1) Add p to seen and add any of p’s pics that are tagged with who to setPics,
 * (2) Recursively do the same with all of p’s pals. */
public static void gather(Person p, String who, Set<Pic> setPics, Set<Person> seen) {
```
(c) (6 points) Write the body of method `getMINE`, below, which solves the overall problem for which we want a method: Find all pictures tagged with a person’s name that that user’s pals, their pals, etc. have. Use the method of part (b).

Note: Person p’s own photos should not be included!

```java
/** Return the set of all pics that p’s pals, their pals, their * pals, etc. have that are tagged with ps name. */
public static Set<Pic> getMINE(Person p) {
}
```

3. **Algorithms** (14 points)

(a) **Sorting** (4 points)

We want to sort an array of \((x, y)\) pairs in lexicographic order, i.e. sort by the \(x\) dimension and break ties with the \(y\) dimension. E.g.: these pairs are in order: \((1, 4), (1, 5), (2, 3)\).

We do this by making two calls to existing function `DimSort(pair-array, dimension)`, which sorts a pair-array in place by an indexed dimension:

- `DimSort(pairs, 1); // sort by y dimension`
- `DimSort(pairs, 0); // sort by x dimension`

What assumption about `DimSort` is needed to make this work?
(b) **Partition algorithm (10 points)**

A variant of the loop of the partition algorithm of quicksort is given by the following precondition, postcondition, and loop invariant.

<table>
<thead>
<tr>
<th>Precondition:</th>
<th>$b$</th>
<th>$? \leq x \geq x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Postcondition 1: $b$</td>
<td>$\leq x \geq x$</td>
<td></td>
</tr>
<tr>
<td>Invariant:</td>
<td>$b$</td>
<td>$\leq x \geq x$</td>
</tr>
</tbody>
</table>

(i) **8 points** Write a loop with initialization that swaps the values of $b[h..k]$ and stores a value in $j$ to truthify the postcondition. Your loop must use the given invariant.

(ii) **2 points** What statement is needed after the loop to truthify the final postcondition shown below?
4. **Data Structures** (28 points)

(a) **Heaps** (6 points)

Array segment b[0..size-1] is supposed to contain a heap — a tree with no holes that satisfies certain properties, as discussed in the lecture on priority queues and heaps. Write the body of the following function. Recursion is not necessary and makes it harder.

```java
/** Return -1, 0, or 1 depending on whether b[0..size-1] is a min-heap,
 * not a heap, or a max-heap.
 * Precondition: size > 1. All the elements of b[0..size-1] are different.
 * b[0..size-1] represents a complete tree, with no holes. */
public static int isHeap(int[] b, int size) {
    // Body of the function
}
```

(b) **Complexity** (7 points)

Consider an undirected connected graph with \( n \) nodes and \( e \) edges. In the implementation, each node is an object of class `Node`, and the neighbors of a node are given as an `ArrayList<Node>`.

Look at method `visit`, below; each statement has a label on it, so we can talk about it.

```java
** Visit all nodes reachable from u along unvisited paths.
 * Precondition. v contains all visited nodes. */
public static void visit(Node u, HashSet<Node> v) {
    a: if (v.contains(u))
    b: return;
    c: v.add(u);
    d: for (Node node : u.neighbors)
    e: visit(node, v);
}
```

Suppose `visit` is called as follows, where `node` is one of the nodes of the graph.

(1) `visit(node, new HashSet<Node>())`;
1. To the right of each of the 5 statements in the body of visit, write the total number of times it will be executed during execution of call (1) above. For (d) we want the number of times the for-each statement is executed, not how many iterations it does.

2. What is the expected time to evaluate the condition of statement (a) once?

3. What would be the expected time to evaluate the condition of statement (a) once if \( v \) were implemented as an ArrayList?

(c) A data structure for words (15 points)

The tree to the right illustrates a neat data structure for maintaining a set of words in the English language. Assume all letters are in lower case. This tree contains the words “to”, “tab”, “no”, and “not”. A word in the set is found by reading off the characters on a path going down to a black node.

Each node has a 26-element array \( b[0..25] \) for the children, each of which represents a lowercase letter and is either null or a (pointer to) a child. Each node also has a bit to say whether that node ends a word or not (black or white in the tree to the right).

(1) 3 points  For the children of a node, \( b[0] \) is for character ‘a’, \( b[1] \) for ‘b’, etc. Let char variable \( c \) contain a lowercase letter. Below, fill in the index so that the array reference returns the value of the b-element for character \( c \):

\[
b[c] \]

(2) 12 points  In the table below, give the tightest expected and worst-case Big-O complexity bounds for searching for a word of length \( n \) in a set of size \( s \) of Strings when the set is implemented as (1) a tree, as above, (2) a Binary Search Tree (BST) of Strings, and (3) a HashSet<String>, remembering that the hash code for a String depends on the length of the String.

<table>
<thead>
<tr>
<th></th>
<th>expected time</th>
<th>worst-case time</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Tree explained above</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) balanced Binary Search Tree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) HashSet&lt;String&gt;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Graphs (12 points)

(a) Shortest-path algorithm (6 points)

State the three-part invariant of Dijkstra’s shortest-path algorithm, involving a settled set, a frontier set, a far-off set, and an array d.

(b) Spanning Trees (6 points)

In introducing the notion of a spanning tree of an undirected connected graph, we stated two properties:

(1) A spanning tree of a graph is a maximal set of edges that contains no cycle.

(2) A spanning tree of a graph is a minimal set of edges that connects all nodes.

Based on (1), we wrote this abstract algorithm for creating a spanning tree:

(A1) While there is a cycle, pick an edge of the cycle and throw it out.

(i) 2 points  Below, give the abstract algorithm that results from using property (2):

(A2)

(ii) 4 points  Each of the abstract algorithms (1) and (2) has a loop. Write down the number of iterations each loop takes (as a function of the number n of nodes and the number e of edges). Based on these number of iterations, state in general which of these two algorithms is preferred.
6. Concurrency (8 points)

(a) Deadlock (4 points)

(1) Define the notion of deadlock. To help you remember, to the right is an image from a lecture slide.
(2) State the standard way of avoiding deadlock.

(b) Producer-consumer problem (4 points)

Recall the producer-consumer problem: Producers (e.g. a bakery chef) place products (e.g. bread loaves) at the end of a bounded queue; consumers (e.g. customer buying bread) take loaves of bread from the front of the queue. Problems arise when the queue is full or empty.

Below is the outline of the bounded-buffer class, with method `produce` specified and partially filled in. Complete the method by filling in the two lines numbered (1) and (2).

```java
/** An instance maintains a bounded buffer of limited size */
public class BoundedBuffer {
    ArrayQueue aq; // bounded buffer implemented in aq

    /** Put v into the bounded buffer. */
    public synchronized void produce(Integer v) {
        while (aq.isFull()) {
            (1) __________________________
        }
        aq.put(v);
        (2) __________________________
    }
```