Object-oriented programming and data-structures

CS/ENGRD 2110
SUMMER 2018
Recall: Data Structures

- List (ArrayList, LinkedList)
- Set (HashSet, TreeSet)
- Map (HashMap, TreeMap)
- Queue (LinkedList)
- PriorityQueue
Recall: Data Structures

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Different abstract data-structures expose different functionality

Set:
- add(E e)
- contains(E e)
- remove(E e)

List:
- add(E e)
- add(int i, E e)
- remove(int i)
- get(int i)
- contains(E e)
Recall: Data Structures

- List (ArrayList, LinkedList)
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- Map (HashMap, TreeMap)
- Queue (LinkedList)
- PriorityQueue

Different implementations have different complexity

HashSet:
- `add(E e)` $O(1)$
- `contains(E e)` $O(1)$
- `remove(E e)` $O(1)$

TreeSet:
- `add(E e)` $O(lg n)$
- `contains(E e)` $O(lg n)$
- `remove(E e)` $O(lg n)$
Priority Queues

- Priority Queues allow you to receive the “next” element in the queue efficiently
  - Where each element has a priority order (or key)
    - (ex: Could be defined by compareTo() in Java)

- Two types of priority queues:
  - Min-Queues
  - Max-Queues
Max Priority Queues

- Supports the following operations:
  - `insert(e, k)` inserts the element e into the queue
  - `maximum()` returns the element with the largest key
  - `extract-max()` removes and returns the element with the largest key
  - `increase-key(e, k)` increases the value of element e’s key to the new value k, which is assumed to be at least as large as e’s current key value
Min Priority Queues

- Supports the following operations:
  - `insert(e,k)` inserts the element `e` into the queue
  - `minimum()` returns the element with the largest key
  - `extract-min()` removes and returns the element with the largest key
  - `decrease-key(e,k)` decreases the value of element `e`'s key to the new value `k`, which is assumed to be smaller than `e`'s current key value
Why priority queues

- Used for event-driven simulations (Emergencies, casualties)
- Graph searching (Dijkstra’s algorithm ...)
- Operating Systems (Load balancing, interrupts)
- Video games
- AI algorithms (A* search algo)
- Compression (Huffman Coding)
interface PriorityQueue<E> {
    boolean add(E e) {...} //insert e.
    E poll() {...} //remove/return min elem.
    E peek() {...} //return min elem.
    void clear() {...} //remove all elems.
    boolean contains(E e)
    boolean remove(E e)
    int size() {...}
    Iterator<E> iterator()
}
Can we implement priority queues?

- Queues are an abstract data type
- Can we already implemented them using what we’ve learnt?
Can we implement priority queues?

- What about a linked list?
Can we implement priority queues?

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  - add() - put new element at the front  \( O(1) \)
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  - poll() - must search the list - $O(n)$
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Can we implement priority queues?

- What about a linked list?
  - `add()` - put new element at the front \(O(1)\)
  - `poll()` - must search the list \(O(n)\)
  - `peek()` - must search the list \(O(n)\)

- What about an **ordered** list?
Can we implement priority queues?

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  - poll() - min element at front - $O(1)$
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- What about an ordered list?
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  - poll() - min element at front - $\mathcal{O}(1)$
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- What about a red-black tree?
Can we implement priority queues?

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  - add() - put new element at the front $O(1)$
  - poll() - must search the list - $O(n)$
  - peek() - must search the list - $O(n)$

- What about an **ordered** list?
  - add() - must search the list - $O(n)$
  - poll() - min element at front - $O(1)$
  - peek() - min element at front - $O(1)$

- What about a **red-black tree**?
  - add()/poll()/peek() - must search the tree & rebalance $O(\log n)$
Can we do better?

- **Goals:**
  - efficiently find the head of the queue
    - Can we do constant time?
  - efficiently insert an element in the queue

- **Non-goals:**
  - find an element that is not the head of the queue
Introducing Heaps

- A heap is a binary tree that satisfies two properties
  - Completeness. Every level of the tree (except last) is completely filled.

  - Heap Order Invariant.
    - Every element in the tree is
      - Smaller or equal than its parent (min-heap)
      - Greater or equal than its parent (max-heap)
Introducing Heaps

- A **heap** is a binary tree that satisfies two properties
  - **Completeness**: Every level of the tree (except last) is completely filled.
  - **Heap Order Invariant**.
    - Every element in the tree is
      - Smaller or equal than its parent (**max-heap**)
      - Greater or equal than its parent (**min-heap**)

Do not confuse with heap memory, where a process dynamically allocates space—different usage of the word heap.
Completeness Property

Every level (except last) completely filled.

Nodes on bottom level are as far left as possible.
Completeness Property

Not a heap because:
Not a heap because:

- missing a node on level 2
Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible
Order Property

Every element is $\leq$ its parent
Every element is $\leq$ its parent

Note: Bigger elements can be deeper in the tree!
Heap Quiz

Which of the following are valid heaps?

(a) 

(b) 

(c) 

(d)
Heap Quiz

Which of the following are valid heaps?

(a) No
(b) No
(c) No
(d) Yes
A **heap** is a binary tree that satisfies two properties

1) Completeness. Every level of the tree (except last) is completely filled. All holes in last level are all the way to the right.

2) Heap Order Invariant. Every element in the tree is <= its parent

A heap implements three key methods:

3) `add(e)`: adds a new element to the heap

4) `poll()`: deletes the max element and returns it

5) `peek()`: returns the max element
add(e)
1. Put in the new element in a new node (leftmost empty leaf)
add(e)

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2. Bubble new element up while greater than parent
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Complexity?
add(e)

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2. Bubble new element up while greater than parent

O(log n)
poll(e)
1. Save root element in a local variable
poll(e)

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2. Assign last value to root, delete last node.
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\[ O(\log n) \]
1. Return root value
peek(e)

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O(1)
Implementing Heaps

public class HeapNode<E> {
    private E value;
    private HeapNode left;
    private HeapNode right;
    ...
}
public class HeapNode<E> {
    private E value;
    private HeapNode left;
    private HeapNode right;
    ...
}

But remember that heaps are complete trees, we can do better!
public class HeapNode<E> {
    private E[] value;
    ...
}

We can use arrays!
Numbering the nodes
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Number node starting at root row by row, left to right
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0 55

1 38

2 22

3 35

4 12

5 19

6 21

20 6 4
Numbering the nodes

Number node starting at root row by row, left to right

Level-order traversal
Numbering the nodes

Number node starting at root row by row, left to right

Level-order traversal

Children of node $k$ are nodes $2k+1$ and $2k+2$
Parent of node $k$ is node $(k-1)/2$
Storing a heap in an array

- Store node number i in index i of array b
- Children of b[k] are b[2k + 1] and b[2k + 2]
- Parent of b[k] is b[(k-1)/2]
add() --assuming there is space
```java
/** An instance of a heap */
class Heap<E> {
    E[] b = new E[50];  // heap is b[0..n-1]
    int n = 0;         // heap invariant is true

    /** Add e to the heap */
    public void add(E e) {
        b[n] = e;
        n = n + 1;
        bubbleUp(n - 1);  // given on next slide
    }
}
```
add() -- BubbleUp
class Heap<E> {
    /** Bubble element #k up to its position.
     * Pre: heap inv holds except maybe for k */
    private void bubbleUp(int k) {
        int p = (k-1)/2
        // inv: p is parent of k and every element
        // except perhaps k is <= its parent
        while (k > 0 && b[k].compareTo(b[p]) > 0) {
            swap(b[k], b[p]);
            k= p;
            p= (k-1)/2;
        }
    }
}
poll()
/** Remove and return the largest element  
  * (return null if list is empty) */

public E poll() {
    if (n == 0) return null;
    E v = b[0];  // largest value at root.
    b[0] = b[n]; // element to root
    n = n - 1;  // move last
    bubbleDown(0);
    return v;
}
poll()
/** Tree has n node.
 * Return index of bigger child of node k
 * (2k+2 if k >= n) */

public int biggerChild(int k, int n) {
    int c = 2*k + 2; // k’s right child
    if (c >= n || b[c-1] > b[c])
        c = c-1;
    return c;
}
poll()
/** Bubble root down to its heap position.  
  Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
    int k = 0;
    int c = biggerChild(k,n);
    // inv: b[0..n-1] is a heap except maybe b[k] AND
    //      b[c] is b[k]'s biggest child
    // inv: b[0..n-1] is a heap except maybe b[k] AND
    //      b[c] is b[k]'s biggest child
    while (c<n && b[k] < b[c] ) {
        swap(b[k], b[c]);
        k = c;
        c = biggerChild(k,n);
    }
}
/** Return the largest element 
 * (return null if list is empty) */
public E peek() {
    if (n == 0) return null;
    return b[0];   // largest value at root.
Let's try it!

Here's a heap, stored in an array:  \[9 5 2 1 2 2]\n
What is the state of the array after execution of add(6)? Assume the existing array is large enough to store the additional element.

A.  \[9 5 2 1 2 2 6]\nB.  \[9 5 6 1 2 2 2]\nC.  \[9 6 5 1 2 2 2]\nD.  \[9 6 5 2 1 2 2\]
Here's a heap, stored in an array:

\[ [9 \ 5 \ 2 \ 1 \ 2 \ 2] \]

Write the array after execution of `add(6)`
Let's try it!

Here's a heap, stored in an array:

\[[9 \ 5 \ 2 \ 1 \ 2 \ 2]\]

Write the array after execution of add(6)
Here's a heap, stored in an array:
\[[9 \ 5 \ 2 \ 1 \ 2 \ 2]\]
Write the array after execution of \text{add}(6)

\[\Rightarrow \ [9 \ 5 \ 6 \ 1 \ 2 \ 2 \ 2]\]
Can we use a heap to sort an array?
Can we use a heap to sort an array?

We said that heaps weren’t great for “finding” an element in the array arbitrarily.

But they’re pretty good at finding the minimum/maximum.

What can we do?
Heap Sort

- Create a heap of the n elements in the array
- Repeatedly extract the minimum (or maximum) until the heap is empty
Heap Sort

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- What’s the cost of creating a heap consisting of n elements?
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  - Equivalent to the cost of inserting n elements into a heap
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  - $n \times \log(n)$
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- What’s the cost of extracting the minimum n times?
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- What’s the cost of extracting the minimum n times?
  - \( n \times \log(n) \)
Heap Sort

- Create a heap of the n elements in the array
- Repeatedly extract the minimum (or maximum) until the heap is empty

What’s the cost of creating a heap consisting of n elements?
- Equivalent to the cost of inserting n elements into a heap
- $n \times \log(n)$

What’s the cost of extracting the minimum n times?
- $n \times \log(n)$

So $n \times \log(n) + n \times \log(n)$: Heapsort is $O(\log n)$!