Final Exam

CS 2110, December 12, 2015, 2:00 PM

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
<th>2</th>
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<th>5</th>
<th>6</th>
<th>Total</th>
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<tbody>
<tr>
<td></td>
<td>True</td>
<td>False</td>
<td>Short Answer</td>
<td>Searching</td>
<td>Sorting</td>
<td>Trees</td>
<td>Graphs</td>
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<td>Max</td>
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The exam is closed book and closed notes. Do not begin until instructed.

You have 150 minutes. Good luck!

Write your name and Cornell NetID at the top of every page! There are 6 questions on 13 numbered pages, front and back. Check that you have all the pages. When you hand in your exam, make sure your booklet is still stapled together. If not, please use our stapler to reattach all your pages!

We have scrap paper available. If you do a lot of crossing out and rewriting, you might want to write code on scrap paper first and then copy it to the exam, so that we can make sense of what you handed in.

Write your answers in the space provided. Ambiguous answers will be considered incorrect. You should be able to fit your answers easily into the space provided.

In some places, we have abbreviated or condensed code to reduce the number of pages that must be printed for the exam. In others, code has been obfuscated to make the problem more difficult. This does not mean that its good style.

**Academic Integrity Statement:** I pledge that I have neither given nor received any unauthorized aid on this exam.

(signature)
1. True / False (20 points)

Circle T or F in the table below.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>a)</td>
<td>T</td>
<td>F</td>
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<td>b)</td>
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<td>F</td>
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<td>T</td>
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<td>r)</td>
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<td>s)</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>t)</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>
2. **Short Questions** (20 points)

2.a **Parsing** (4 points)

The following grammar describes numbers in scientific notation. Spaces are not significant. \(<sci>\) is the start symbol of the grammar.

\[
\begin{align*}
<\text{nz digit}> & \rightarrow \ 1 \ | \ 2 \ | \ 3 \ | \ 4 \ | \ 5 \ | \ 6 \ | \ 7 \ | \ 8 \ | \ 9 \\
<\text{digit}> & \rightarrow \ 0 \ | \ <\text{nz digit}> \\
<\text{int}> & \rightarrow \ <\text{digit}> \ | \ <\text{digit}><\text{int}> \\
<\text{exp}> & \rightarrow \ E+<\text{int}> \ | \ E-<\text{int}> \\
<\text{sci}> & \rightarrow \ <\text{nz digit}>.<\text{int}><\text{exp}> \ | \ -<\text{nz digit}>.<\text{int}><\text{exp}> \ | \ 0.0
\end{align*}
\]

Indicate which of the following statements are true and which are false.

(i) The following is a sentence of the grammar: \(1.0618E-15\)

(ii) The following is a sentence of the grammar: \(9.0\)

(iii) The number of sentences of the grammar is infinite.

(iv) This is a \(<\text{digit}>\): \(01\)

2.b **Testing** (4 points)

Consider function `findMedian`. (If \(b.length\) is odd, the median is the middle value when \(b\) is sorted; if \(b.length\) is even, the median is the average of the two middle values when \(b\) is sorted.)

```java
/** Return the median of the numbers in b. Precondition: b.length > 0. */
public static double findMedian(int[] b) {...}
```

Complete procedure `testFindMedian` below, filling in enough test cases to have reasonable confidence that `findMedian` is correct.

```java
@Test
public void testFindMedian() {
}
```
2.c Exception handling (4 points)

Function `Integer.parseInt(String s)` returns the int value of the integer that is in `s`. But if `s` does not contain an integer, `parseInt` throws a `NumberFormatException`. Write a statement that stores in variable `ans` the value of the function call `Integer.parseInt(someString)` but stores 1 in `ans` if a `NumberFormatException` is thrown.

2.d Classes and Interfaces (4 points)

Consider the following definitions and interfaces:

```java
interface I1 {}
interface I2 {}
class A implements I2 {}
class B implements I1 {}
class C extends B implements I2 {}
...
A ob1= ...;
B ob2= ...;
C ob3= ...;
```

Say whether each of the following statements is legal or illegal. Consider each statement by itself.

(i) `ob2= ob3;`
(ii) `ob3= ob2;`
(iii) `I1 b= ob3;`
(iv) `I2 c= ob1;`
2.e Complexity (5 points)

(i) (2 points) Write the definition of “$f(n)$ is $O(g(n))$”.

(ii) (3 points) Complete the following statements (giving the tightest bound on the $O(...)$ complexity):

1. Worst-case time for quicksort on an array of size $n$ is:
2. Worst-case time for testing membership in a hash set of size $n$ is:
3. Expected time for testing membership in a hash set of size $n$ when load factor is $1/2$ is:
3. Searching / Sorting (15 points)

3.a Loop invariants (7 points)

Suppose we are given an array segment \( b[0..n] \) with \( 0 \leq n \). The array segment is sorted in ascending order but possibly contains duplicates. We want to move the elements around so that all of the values in the original array segment are in array segment \( b[k..n] \) with duplicates removed and still in ascending order. For example, given array \( b[0..n] \) on the left below, we want to produce the array on the right, where \( b[k..n] \) contains the values with duplicates removed.

We don’t care what is in \( b[0..k - 1] \).

\[
\begin{array}{cccccccc}
& 0 & & n & & 0 & & k & & n \\
\hline
b & 2 & 2 & 4 & 4 & 4 & 5 & 8 & 8 & 2 & 2 & 4 & 4 & 2 & 4 & 5 & 8 \\
\end{array}
\]

Below are a precondition, postcondition for the problem and a loop invariant. “Don’t care” means we do not care what values are in that segment; “no dups” means “no duplicates”.

<table>
<thead>
<tr>
<th>Precondition:</th>
<th></th>
<th>original ( b[0..n] )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>( k )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Postcondition:</th>
<th></th>
<th>don’t care</th>
<th>original ( b[0..n] ), no dups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>( k )</td>
<td>( n )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Invariant:</th>
<th></th>
<th>original ( b[t..t] )</th>
<th>don’t care</th>
<th>original ( b[t+1..n] ), no dups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>( k )</td>
<td>( n )</td>
<td></td>
</tr>
</tbody>
</table>

Below, write a loop with initialization for this problem. Your grade depends on how well you use the given loop invariant and the four loopy questions. Hint: \( b[0..n] \) contains at least one value, so think of starting with \( b[k..n] \) containing one value.
3.b Selection Sort (2 points)

Write the invariant for `selectionSort` method.

```java
/** Sort b --put its elements in ascending order. */
public static void selectionSort(Comparable[] b) {
    // Invariant:
    ...
}
```

3.c Merge Sort (6 points)

The code for merge sort is given below. It contains errors. Fix the errors. Assume that method `merge` has the specification shown after the method.

```java
/** Sort b[h..k]. */
public static void mergeSort(Comparable[] b, int h, int k) {
    if (h < k) return;
    int e = h / 2;
    merge(b, h, e, k);
    mergeSort(b, h, e);
    mergeSort(b, e, k);
}

/** b[h..j] and b[j+1..k] are sorted.
* Merge them together so that b[h..k] is sorted. */
public static void merge(int[] b, int h, int j, int k) {...}
```
4. **Trees** (15 points)

A *trie* is a tree data structure that can be used to implement a set of *String* values. Each node in a trie may have multiple children, which are unordered, and an edge between a given parent and child is annotated with a character. Each element in the set can be found by reading off the characters on some path going from the root to a leaf. For example, here is a trie containing three strings: “camel”, “corn”, and “cows”

![Trie Diagram]

A trie consisting of a single node represents the empty string “”.

4.a **Warmup** (5 points)

Draw a trie containing the following entries (note that ‘F’ and ‘f’ are different characters):

- “Fred”
- “fried”
- “frozen”
- “food”
- “Friday”
4.b Implementation (10 points)

In real life, “for” and “form” are both words, which would be allowed in a trie. However, to keep things simple, in our implementation, only strings given by a path from the root to a leaf are considered to be in the set. Consider the following partial implementation of a trie:

```java
/* An instance is the root of a trie that represents a set of strings */
public class Trie {
    /* A key-value pair (c, t) represents the edge labeled with *
     * character c from this trie to trie t. */
    private Map<Character, Trie> children;

    /** Constructor: an empty set */
    public Trie() {
        children = new HashMap<Character, Trie>();
    }

    ...}
```

Complete the definition of method `insert`, which of course goes in class `Trie`. *Hint: use recursion and don’t forget the base case.*

```java
/** Insert s into this trie. *
 * Precondition: s is not a proper substring of some string in the trie. */
public void insert(String s) {
```
5. **Graphs** (23 points)

(a) 5 points  Consider the following graph:

List the edges of the graph above in the order they would be added to a minimum spanning tree by Prim’s algorithm. If a starting node is required, start with node $a$. An edge should be listed by the pair of vertices it joins. For example, the edge with weight 15 above would be edge $(e, f)$.
(b) 7 points  Consider the following graph:

Execute Dijkstra’s shortest-path algorithm on the graph above with a as the start node. We show the initial state, before iteration 0 of the main loop, in the second column below, giving the frontier set (the blue set in the description of Dijkstra’s algorithm given in lecture), and the $L$ value for each node so far.

For each iteration 0, 1, 2, ..., first write in the appropriate place in the table the Selected node, that is, the node that gets removed from the frontier set. Below that, write the new value of the frontier set and the $L$ values that change on this iteration.

<table>
<thead>
<tr>
<th>Selected Node</th>
<th>N/A</th>
<th>Frontier Set</th>
<th>{$a$}</th>
<th>$L[a]$</th>
<th>0</th>
<th>$L[b]$</th>
<th>$\infty$</th>
<th>$L[c]$</th>
<th>$\infty$</th>
<th>$L[d]$</th>
<th>$\infty$</th>
<th>$L[e]$</th>
<th>$\infty$</th>
<th>Iteration</th>
<th>Init</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Final</th>
</tr>
</thead>
</table>
(c) 11 points  Consider the following class representing a node of an undirected graph that has an infinite number of nodes.

```java
class Node {
    Set<Node> neighbors; // Neighbors of this node
    boolean hasTreasure; // true iff this Node has a treasure on it
}
```

Complete function `findTreasure` below according to its specification. “REACHABLE” means reachable along a path of unvisited nodes.

```java
/** Return a node REACHABLE from n that has a treasure on it.  
* Precondition: at least one node REACHABLE from n has a treasure on it  
* and that node is a finite distance from n */
public static Node findTreasure(Node n) {
    HashSet<Node> visited = new HashSet<Node>(); // Nodes that have been visited
    LinkedList<Node> queue = new LinkedList<Node>();
    queue.add(n);
    // Invariant: a REACHABLE node with a treasure is REACHABLE from
    // some unvisited node in queue
    while ( ) { // Fill in loop condition
        // Complete loop body
        return null; // Never executed, keeps Java happy
    }
}
```
6. **Concurrency** (7 points)

This question deals with the bounded buffer problem, but pared down to the bare minimum. Consumers (each a thread) can get an OK to do something by calling function `consume` (see below). A system is introduced to be able to slow down the rate at which OKs are given. A bunch of producers (threads) call procedure `produce` when they want; each call on `produce` allows one more consumer to get an OK. At any time, at most 10 consumers can get an OK.

```java
class OK {
    int n; // 0 <= n <= 10. The number of consumers who may get an OK.
    // n is increased by producers, decreased by consumers

    /** Increase the number of consumers who can get an OK, but
     * don't let it get over 10. */
    public synchronized void produce() {
        while (n == 10) wait(); // wait until n < 10
        n= n+1;
        notifyAll(); // Signal to all that an OK may be granted
    }

    /** Return "OK" but wait until it is allowed */
    public synchronized String consume() {
        if (n == 0) wait(); // wait until n != 0
        n= n-1;
        notifyAll(); // Signal to all that an "OK" has been granted: n < 10
        return "OK";
    }
}
```

(a) **3 points** Unfortunately, the code is not thread safe, and a bunch of calls involving just 2 consumers, \(c_1\) and \(c_2\), and one producer, \(p\), can lead to \(n\) becoming negative. Describe the sequence of calls and show why \(n\) becomes negative.

(b) **4 points** Fix class `OK` to eliminate the error you found in (a).