Abstract vs concrete data structures

- Abstract data structures are **interfaces**
  - they specify only **interface** (method names and specs)
  - not **implementation** (method bodies, fields, …)

- Concrete data structures are **classes**. Abstract data structures can have multiple possible implementations by different concrete data structures.
## Abstract vs concrete data structures

- **interface** List defines an “abstract data type”.
- It has methods: add, get, remove, ...
- Various **classes** ("concrete data types") implement List:

<table>
<thead>
<tr>
<th></th>
<th>ArrayList</th>
<th>LinkedList</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Backning storage:</strong></td>
<td>array</td>
<td>chained nodes</td>
</tr>
<tr>
<td><strong>add(i, val)</strong></td>
<td>$O(n)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td><strong>add(0, val)</strong></td>
<td>$O(n)$</td>
<td>$O(1)$</td>
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<tr>
<td><strong>add(n, val)</strong></td>
<td>$O(1)$</td>
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<tr>
<td><strong>get(i)</strong></td>
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<td>$O(n)$</td>
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<td>$O(1)$</td>
<td>$O(1)$</td>
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</tbody>
</table>
4 Concrete Data Types
Concrete data structures

- Array
- LinkedList (singly-linked, doubly-linked)
- Trees (binary, general)
- Heaps
Heaps

- A *heap* is a binary tree with certain properties (it's a concrete data structure)
  - Heap Order Invariant: every element in the tree is >= its parent
  - Complete Binary Tree: every level of the tree (except last) is completely filled, there are no holes

Do not confuse with *heap memory*, where the Java virtual machine allocates space for objects — different usage of the word *heap*
Every element is $\geq$ its parent

Note: 19, 20 < 35: Smaller elements can be deeper in the tree!
Completeness Property

Every level (except last) completely filled.

Nodes on bottom level are as far left as possible.
Not a heap because it has two holes

Not a heap because:

- missing a node on level 2
- bottom level nodes are not as far left as possible
Heaps

• A heap is a binary tree with certain properties (it's a concrete data structure)
  • Heap Order Invariant: every element in the tree is \( \geq \) its parent
  • Complete Binary Tree: every level of the tree (except last) is completely filled, there are no holes
• A heap implements two key methods:
  • add(e): adds a new element to the heap
  • poll(): deletes the least element and returns it
add (e)
1. Put in the new element in a new node
add()

2. Bubble new element up if less than parent
add()

2. Bubble new element up if less than parent
add()
add(e)

• Add e at the leftmost empty leaf
• Bubble e up until it no longer violates heap order
• The heap invariant is maintained!
add(e) to a tree of size \( n \)

- Time is \( O(\log n) \), since the tree is balanced
  - size of tree is exponential as a function of depth
  - depth of tree is logarithmic as a function of size
poll()
poll()

1. Save top element in a local variable
2. Assign last value to the root, delete last value from heap
poll()

3. Bubble root value down
poll()

3. Bubble root value down
poll()

3. Bubble root value down
poll()

- Save the least element (the root)
- Assign last element of the heap to the root.
- Remove last element of the heap.
- Bubble element down – always with smaller child, until heap invariant is true again.

The heap invariant is maintained!

- Return the saved element

Time is O(log n), since the tree is balanced
public class HeapNode<E> {
    private E value;
    private HeapNode left;
    private HeapNode right;
    ...
}
public class Heap<E> {
    private E[] heap;
    ...
}
Numbering the nodes in a heap

Number node starting at root row by row, left to right

Level-order traversal

Children of node $k$ are nodes $2k+1$ and $2k+2$
Parent of node $k$ is node $(k-1)/2$
Store a heap in an array (or ArrayList) $b$!

- Heap nodes in $b$ in order, going across each level from left to right, top to bottom
- Children of $b[k]$ are $b[2k + 1]$ and $b[2k + 2]$
- Parent of $b[k]$ is $b[(k - 1)/2]$

Tree structure is implicit. No need for explicit links!
/** An instance of a heap */
class Heap<E> {
    E[] b = new E[50]; // heap is b[0..n-1]
    int n = 0; // heap invariant is true

    /** Add e to the heap */
    public void add(E e) {
        b[n] = e;
        n = n + 1;
        bubbleUp(n - 1); // given on next slide
    }
}
add(). Remember, heap is in b[0..n-1]

class Heap<E> {
    /** Bubble element #k up to its position.
     * Pre: heap inv holds except maybe for k */
    private void bubbleUp(int k) {
        int p = (k-1)/2;
        // inv: p is parent of k and every elmnt
        // except perhaps k is >= its parent
        while (k > 0 && b[k].compareTo(b[p]) < 0) {
            swap(b[k], b[p]);
            k = p;
            p = (k-1)/2;
        }
    }
}
poll(). Remember, heap is in b[0..n-1]

/** Remove and return the smallest element  
  * (return null if list is empty) */
public E poll() {
    if (n == 0) return null;
    E v = b[0]; // smallest value at root.
    n = n - 1; // move last
    b[0] = b[n]; // element to root
    bubbleDown(0);
    return v;
}
c’s smaller child

```java
/** Tree has n node.
 * Return index of smaller child of node k
 * (2k+2 if k >= n) */

public int smallerChild(int k, int n) {
    int c = 2*k + 2;  // k’s right child
    if (c >= n || b[c-1].compareTo(b[c]) < 0)
        c = c-1;
    return c;
}
```
/** Bubble root down to its heap position.  
Pre: b[0..n-1] is a heap except maybe b[0] */
private void bubbleDown() {
    int k = 0;
    int c = smallerChild(k, n);
    // inv: b[0..n-1] is a heap except maybe b[k] AND
    // b[c] is b[k]'s smallest child
    while (c < n && b[k].compareTo(b[c]) > 0) {
        swap(b[k], b[c]);
        k = c;
        c = smallerChild(k, n);
    }
}
Abstract Data Types
Some Abstract Data Types

• List

• Stack (LIFO) implemented using a List
  - allows only \texttt{add(0, val)}, \texttt{remove(0)} (push, pop)

• Queue (FIFO) implemented using a List
  - allows only \texttt{add(n, val)}, \texttt{remove(0)} (enqueue, dequeue)

Both efficiently implementable using a singly linked list with head and tail

• PriorityQueue
Priority Queue

• Data structure in which data items are Comparable

• Smaller elements (determined by compareTo()) have higher priority

• remove() return the element with the highest priority = least element in the compareTo() ordering

• break ties arbitrarily
Many uses of priority queues (& heaps)

- Event-driven simulation: customers in a line
- Collision detection: "next time of contact" for colliding bodies
- Graph searching: Dijkstra's algorithm, Prim's algorithm
- AI Path Planning: A* search
- Statistics: maintain largest $M$ values in a sequence
- Operating systems: load balancing, interrupt handling
- Discrete optimization: bin packing, scheduling
- College: prioritizing assignments for multiple classes.

Surface simplification [Garland and Heckbert 1997]
interface PriorityQueue<E> {
    boolean add(E e) {...} //insert e.
    void clear() {...} //remove all elems.
    E peek() {...} //return min elem.
    E poll() {...} //remove/return min elem.
    boolean contains(E e)
    boolean remove(E e)
    int size() {...}
    Iterator<E> iterator()
}

IF implemented with a heap!
• Maintain as a list
  – add() put new element at front – O(1)
  – poll() must search the list – O(n)
  – peek() must search the list – O(n)

• Maintain as an ordered list
  – add() must search the list – O(n)
  – poll() min element at front – O(1)
  – peek() O(1)

Can we do better?
Priority queues as heaps

• A heap is can be used to implement priority queues
• Gives better complexity than either ordered or unordered list implementation:
  - \texttt{add}(): \( O(\log n) \) (\( n \) is the size of the heap)
  - \texttt{peek}(): \( O(1) \)
  - \texttt{poll}(): \( O(\log n) \)
What if the priority is independent from the value?

Separate priority from value and do this:

\texttt{add(e, p); //add element e with priority p (a double)}

THIS IS EASY!

Be able to change priority

\texttt{change(e, p); //change priority of e to p}

THIS IS HARD!

Big question: How do we find \(e\) in the heap?
Searching heap takes time proportional to its size! No good!
Once found, change priority and bubble up or down. OKAY

Assignment A6: implement this heap! Use a second data structure to make change-priority expected log \(n\) time